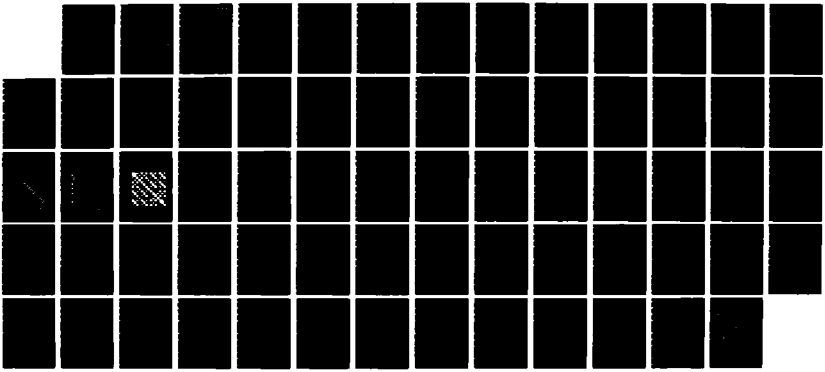
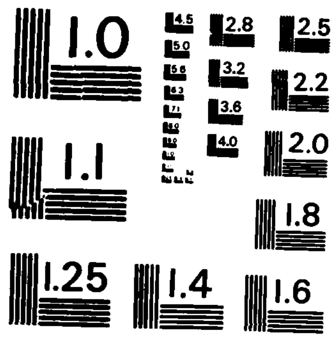
AD-A166 792 AN ECONOMICAL MULTIFACTOR WITHIN-SUBJECT DESIGN ROBUST AGAINST TREND AND CARRYOVER EFFECTS(U) ESSEX CORP ORLANDO FL C W SIMON 17 OCT 85 EOTR-86-2 1/1

UNCLASSIFIED N61339-81-C-0105 F/G 5/10 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)

ESSEX ORLANDO
TECHNICAL REPORT
EOTR 86-2

AN ECONOMICAL MULTIFACTOR WITHIN-SUBJECT DESIGN ROBUST AGAINST TREND AND CARRYOVER EFFECTS

Charles W. Simon

Contract No. N61339-81-C-0105

FINAL REPORT

17 October 1985

Prepared for:

Naval Training Systems Center
Orlando, Florida 32803

Essex Corporation
1040 Woodcock Road
Orlando, Florida 32803

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A166 772

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT DISTRIBUTION STATEMENT A Approved for public release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Essex Corporation 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Naval Training System Center
6c. ADDRESS (City, State, and ZIP Code) 1040 Woodcock Road Orlando, FL 32803	7b. ADDRESS (City, State, and ZIP Code) Orlando, FL 32813
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N61339-81-C-0105
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO. | WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
An Economical Multifactor Within-Subject Design Robust Against Trend and Carryover Effects

12. PERSONAL AUTHOR(S)
Charles W. Simon

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
	FROM ____ TO ____	17 October 1985	63

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP
05	08	
14	02	

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Economical multifactor design, within-subject design, trend-robust design, carryover-robust design, holistic approach, between-subject design, (contd)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

When the same subject is tested serially across the conditions of an experimental design, unless precautions are taken, there is a large chance that the experimental results will be biased by sequence effects. In human performance research, two types of sequence effects are quite common, i.e., trends through the data, such as learning (and even systematic variations in the equipment and/or environment) and carryover effects, when the characteristics of one condition influence the performance on the condition that follows. Unless these effects are removed or neutralized, the information obtained from the experiment will be distorted and incorrect conclusions may be drawn.

Whether 2^k factors or 2^{k-p} fractional factorials are used with the within-subject approach, sequence effects must be properly handled. The

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code) | 22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

19. Key Words: (contd) trend-carryover robust (TCR) design, repeated-treatment design

20. Abstract: (contd)

traditional way of counterbalancing or assigning conditions at random can be inadequate and certainly uneconomical. When large-scale multifactor experiments are performed, concern with economy is paramount.

In this report, a within-subject design constructed from 2^5 factorial plan is evolved into a 2^{7-2} fractional factorial, Resolution IV, design. The novelty of this design is that with limited but reasonable restrictions, direct effects are robust against linear, quadratic, and cubic trend effects and additive and interactive one-trial carryover effects. Tables are provided that will help an investigator make the decisions necessary to properly use the design for screening purposes. The principles by which larger designs can be constructed are provided. Thus, we can screen up to seven factors with 32 serially-tested experimental conditions, 15 factors with 64 conditions, and 31 factors with 128 conditions without serious concern for sequence effects. In some cases, additional conditions may be required in accordance with the holistic approach. Several plans for analyzing the data are suggested.

PER LETTER

A-1

QUALITY INSPECTED 3

TABLE OF CONTENTS

TABLE OF CONTENTS (contd)

LIST OF TABLES

PREFACE

The contributions of the following persons are gratefully acknowledged:

Dr. Daniel P. Westra noted and clarified a serious logico-statistical error that I had made in an early version of this report. He also developed some of the computer programs used in the analyses.

Mr. Ronald Mauk wrote and ran some of the computer programs used in the analyses.

Dr. E. Christopher Poulton, Applied Psychology Unit, Medical Research Council, Cambridge, England, made many valuable suggestions regarding the contents of this report. His comments regarding the inadequate handling of carryover effects in an earlier report of mine was a primary stimulant leading to this investigation.

Mrs. Patricia Patrizzi materially assisted in the preparation of this report.

SECTION I

INTRODUCTION

Too frequently, textbooks on experimental design treat separately the spatial and the temporal assignments of data collection points. Consequently, experimenters more attentive to the former task often fail to reduce or isolate sources of unwanted sequence effects that may severely bias experimental results (Poulton, 1975). In an analysis of 239 experiments published in Human Factors from 1958 to 1972, Simon (1976) found that 31% used a form of the "within-subject" design, in which each subject is tested serially over many experimental conditions. Few of these, however, were designed or analyzed in a way that would offset sequence effects likely to be engendered by the design.

Of the various types of sequence effects that are likely to occur, psychologists have shown the greatest concern for trends associated with operator learning and fatigue. When such effects are unwanted, investigators have traditionally tried to offset them by randomizing or systematically counterbalancing the order in which the experimental conditions are presented to the subject. Unfortunately, these methods have often been inadequate and/or improperly employed. Furthermore, when large scale multifactor experiments are to be designed, these conventional means of handling sequence effects require more replications of the basic design than ordinarily would be economically acceptable.

As an alternative, designs are needed that are economical, yet able to minimize the effects of selected sequence effects. Simon (1977a) describes an economical multifactor 2^{k-p} design of Resolution IV suitable for screening studies that is robust against linear, quadratic, and cubic trend effects with no replicated conditions in the design. In this report, that design is analyzed to determine to what extent it is also robust against carryover effects. A modified design is proposed that is robust against trend and one-trial carryover effects.

WHAT IS A CARRYOVER EFFECT?

A "carryover" effect is said to occur when experience on one experimental condition affects the performance on a different condition that follows it (Simon, 1974). Carryover effects may be simple or complex and may last for one or many

trials. In the statistical and psychological literature, such terms as "change over", "residual", and "transfer" have been used synonymously with "carryover", the term that will be used in this report.

SECTION II

BRIEF REVIEW OF TREND-ROBUST HOLISTIC DESIGNS [1]*

In Table 1, an example of a 2^{16-11} Resolution IV trend-robust holistic design is shown. The basic design is made up of 32 experimental conditions. If fully saturated, it is theoretically possible to examine the effects of 16 different factors (which may include the "trend factors"), and to obtain some indication of the existence of critical two-factor interaction effects. Main effects can be estimated independently of all other main effects and of all two-factor interaction effects, but are confounded with strings of three-factor interactions (Appendix A). Two-factor interactions are aliased in sub-sets of independent strings (Appendix B). As with all fractional factorial designs, the indicated effects are actually aliased with still higher-order effects. Specifically in this design, which is a one-two-thousand-and-forty-eighth fraction of the full factorial design for 16 factors, the main effects are aliased with three-, five-, and all other odd-numbered interactions and the two-factor interactions are aliased with themselves and with all even-numbered interactions.

This aliasing of sources of variance [2] can be tolerated in most experiments because of certain characteristics of human performance data and of the paradigm associated with the holistic approach that differentiates it from the traditional fractional factorial design. Critical intrinsic three-factor interactions are found infrequently in human performance data, at least when the variables are quantitative, and finding a critical intrinsic four-factor interaction effect would be an extremely rare event (Simon, 1971; 1973; 1976). Therefore, these designs can be used with impunity as long as the assumption that higher-order interactions are not likely to exist is employed only as a tentative "working hypothesis" that <u>eventually will be tested</u> as prescribed by the holistic approach.

Economy is an important feature of the holistic approach. Data for the full factorial will probably never be collected because to do so adds no new information beyond a certain level

---

*All future footnotes are presented collectively beginning on page 42. They are noted by the numbers in brackets, e.g., [1].

of complexity. Instead, data is systematically collected a little at a time according to a special paradigm, each new step being taken only when tests indicate that additional data is likely to produce additional information. Some features employed to support the successful application of this heirarchial approach are:

(a) Data are collected only to satisfy the requirements of each step of an informational heirarchy. Major plateaus in the data collection process include: (1) screening many potentially critical factors, (2) developing a response surface based on the critical ones, and (3) obtaining more precise performance data for selected configuration (see Simon, 1977b; 1979).

(b) An elaborate preexperimental analysis is used to try to rationally spotlight potentially critical interactions prior to data collection and to place them within the fractional factorial experimental design where they may best be detected if the assumptions are valid.

(c) Special techniques are employed to detect the lack of fit of linear and/or quadratic models (when continuous data are involved). Data is not collected if these tests fail.

(d) Transformation techniques are employed to try to eliminate higher-order effects. If the model can be linearized through proper scaling, the requirement for additional data may be eliminated.

(e) During screening, critical interactions are important only to the extent they may involve factors that individually did not prove to be critical. When critical interaction strings are detected it may be necessary to collect additional data in a systematic modular fashion to identify which interactions in the string are the critical ones (see Daniel, 1976).

In practice, one seldom uses a saturated experimental design. There are practical reasons why the design in Table 1 -- with its potential for handling 16 factors -- would only be used to investigate no more than approximately 12 factors. One needs the extra degrees of freedom to represent sequence effects and possibly some three-factor interactions.

Note that in Table 1, two sets of labels are provided: (1) new screening design labels, and (2) original factor labels. The first set of labels is used in an economical, multifactor experiment that takes into consideration the fractional nature of the design, while the second set -- the traditional one -- is more useful when thinking about and manipulating the design itself. Thus, the sign matrix for a 2^{k-p} fractional factorial is the same as a 2n factorial when (k-p) = n. However, certain advantages can be realized if the columns are shifted about as shown in Table 1 for reasons to be explained below.

The columns and row of the traditional factorial design are ordinarily written in Yates' standard order. When labelled, this order for the columns would be an expansion of the form (original labels): A, B, AB, C, AC, BC, ABC, D, AD... and so forth, depending on how many factors are involved and a similar pattern for rows: (1), a, b, ab, c, ac ... and so forth. In this standard order, the pattern of minus and plus signs in the factor A column, for example, would be ordered: − + − + − + − + and so forth, where the plus and minus signs are associated with experimental conditions containing the low and high levels of Factor A, respectively (or any two categorical levels). Except for the need to maintain the validity of the generalized interactions, which columns were selected for main effects had never been an issue until Daniel and Wilcoxin (1966) showed how it mattered and how it could be used to reduce the bias from trend effects. Simon (1977a) developed this procedure still further and arrived at the holistic trend-robust design shown in Table 1.

To optimize the fractional factorial so that it will be effectively robust against trend effects, the design in Table 1 is arranged as follows:

(1) All columns with letter A in the original factor labels are used for the main effects. When there are not enough main effects to fill the A-columns, those columns left over represent independent strings of three-factor interactions (Appendix A).

(2) For both main and interaction effects, the columns are ordered (using original factor labels) so that the highest interactions come first, followed by smaller and smaller interactions until the main effects are reached.

(3) Within a series of interactions of the same degree, the interactions are listed alphabetically using the original factorial labels.

The degree to which each column of the design is affected by trend effects is shown at the bottom of Table 1 in the rows labelled: "Percent Trend/Effect Overlap." This percent overlap is the proportion of variance (times 100) of the factor effect that could be attributed to the trend effect. The column rearrangement _per se_ does not affect the degree of trend-robustness of each column, but does facilitate the use of the overall design. For all practical purposes, no main effect (new factor labels) will be seriously affected by a linear, quadratic, and/or cubic trend should any exist. Only five of the 15 two-factor interaction strings will be confounded with trends to any notable effect and of these, only three appear serious enough to avoid. Of course, the importance of this confounding also depends on the relative strength of the trend effects themselves. For example, a 71% trend-factor overlap may be irrelevant if the actual trend effect is trivial, while a

TABLE 1. SIGN MATRIX FOR A 2^{16-11}_{IV} TREND RESISTANT SCREENING DESIGN (N = 32)

TEST ORDER	EXPERIMENTAL CONDITION	NEW SCREENING DESIGN LABELS*																															
		(MAIN EFFECTS)*																						(TWO-FACTOR INTERACTION STRINGS)**									
		(I)	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	AP	AO	AN	AM	AL	AK	AJ	AI	AH	AG	AF	AE	AD	AC	AB
1	BCDELMNO	+	-	+	+	+	+	-	-	-	-	-	-	+	+	+	+	-	+	-	-	-	-	+	+	+	+	+	+	-	-	-	-
2	AFGHIJKP	+	+	-	-	-	-	+	+	+	+	+	+	-	-	-	-	+	+	-	-	-	-	+	+	+	+	+	+	-	-	-	-
3	AEFGHMNO	+	+	-	-	-	+	+	+	+	-	-	-	-	+	+	+	-	-	+	+	+	-	-	-	-	+	+	+	+	-	-	-
4	BCDIJKLP	+	-	+	+	+	-	-	-	-	+	+	+	+	-	-	-	+	-	+	+	+	-	-	-	-	+	+	+	+	-	-	-
5	ADFIJLNO	+	+	-	-	+	-	+	-	-	+	+	-	+	-	+	+	-	-	+	+	-	+	-	+	+	-	-	+	-	+	-	-
6	BCEGHKMP	+	-	+	+	-	+	-	+	+	-	-	+	-	+	-	-	+	-	+	+	-	+	-	+	+	-	-	+	-	+	-	-
7	BCGHIJNO	+	-	+	+	-	-	-	+	+	+	+	-	-	-	+	+	-	+	-	-	+	+	+	-	-	-	-	+	+	+	-	-
8	ADEFKLMP	+	+	-	-	+	+	+	-	-	-	-	+	+	+	-	-	+	+	-	-	+	+	+	-	-	-	-	+	+	+	-	-
9	ACGIKLMO	+	+	-	+	-	-	-	+	-	+	-	+	+	+	-	+	-	-	+	-	+	+	+	-	+	-	+	-	-	-	+	-
10	BDEFHJNP	+	-	+	-	+	+	+	-	+	-	+	-	-	-	+	-	+	-	+	-	+	+	+	-	+	-	+	-	-	-	+	-
11	BDFHIKMO	+	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-
12	ACEGJLNP	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-
13	BEFGJKLO	+	-	+	-	-	+	+	+	-	-	+	+	+	-	-	+	-	+	-	+	+	-	-	-	+	+	-	-	-	+	+	-
14	ACDHIMNP	+	+	-	+	+	-	-	-	+	+	-	-	-	+	+	-	+	+	-	+	+	-	-	-	+	+	-	-	-	+	+	-
15	ACDEHJKO	+	+	-	+	+	+	-	-	+	-	+	+	-	-	-	+	-	-	+	-	-	-	+	+	-	+	-	-	+	+	+	-
16	BFGILMNP	+	-	+	-	-	-	+	+	-	+	-	-	+	+	+	-	+	-	+	-	-	-	+	+	-	+	-	-	+	+	+	-
17	ABHJKLMN	+	+	+	-	-	-	-	-	+	-	+	+	+	+	+	-	-	-	-	+	+	+	+	+	-	+	-	-	-	-	-	+
18	CDEFGIOP	+	-	-	+	+	+	+	+	-	+	-	-	-	-	-	+	+	-	-	+	+	+	+	+	-	+	-	-	-	-	-	+
19	CDFGJKMN	+	-	-	+	+	-	+	+	-	-	+	+	-	+	+	-	-	+	+	-	-	+	-	-	+	+	-	-	+	-	-	+
20	ABEHILOP	+	+	+	-	-	+	-	-	+	+	-	-	+	-	-	+	+	+	+	-	-	+	-	-	+	+	-	-	+	-	-	+
21	CEFHIKLN	+	-	-	+	-	+	+	-	+	+	-	+	+	-	+	-	-	+	+	-	+	-	-	+	-	-	+	-	-	+	-	+
22	ABDGJMOP	+	+	+	-	+	-	-	+	-	-	+	-	-	+	-	+	+	+	+	-	+	-	-	+	-	-	+	-	-	+	-	+
23	ABDEGIKN	+	+	+	-	+	+	-	+	-	+	-	+	-	-	+	-	-	-	-	+	-	-	+	-	+	-	+	-	+	+	-	+
24	CFHJLMOP	+	-	-	+	-	-	+	-	+	-	+	-	+	+	-	+	+	-	-	+	-	-	+	-	+	-	+	-	+	+	-	+
25	DEGHIJLM	+	-	-	-	+	+	-	+	+	+	+	-	+	+	-	-	-	+	+	+	-	-	+	-	-	-	-	+	-	-	+	+
26	ABCFKNOP	+	+	+	+	-	-	+	-	-	-	-	+	-	-	+	+	+	+	+	+	-	-	+	-	-	-	-	+	-	-	+	+
27	ABCEFIJM	+	+	+	+	-	+	+	-	-	+	+	-	-	+	-	-	-	-	-	-	+	-	-	+	+	-	-	+	+	-	+	+
28	DGHKLNOP	+	-	-	-	+	-	-	+	+	-	-	+	+	-	+	+	+	-	-	-	+	-	-	+	+	-	-	+	+	-	+	+
29	ABCDFGHL	+	+	+	+	+	-	+	+	+	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	+	+	+	-	+	+	+
30	EIJKMNOP	+	-	-	-	-	+	-	-	-	+	+	+	-	+	+	+	+	-	-	-	-	+	-	-	-	+	+	+	-	+	+	+
31	(I)	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
32	ABCDEFGHIJKLMNOP	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ORIGINAL FACTOR LABELS		(I)	ABCDE	ABCD	ABCE	ABDE	ACDE	ABC	ABD	ABE	ACD	ACE	ADE	AB	AC	AD	AE	A	BCDE	BCD	BCE	BDE	CDE	BC	BD	BE	CD	CE	DE	B	C	D	E
PERCENT# TREND/EFFECT OVERLAP***	LINEAR																	0												1	5	19	75
	QUADRATIC													0	0	0	1							0	1	4	4	18	71				
	CUBIC							0	0	0	0	1	2					0		1	2	10	39							3	9	21	11
FACTOR LEVEL CHANGE COUNT		0	21	20	22	18	26	23	19	17	27	25	29	16	24	28	30	31	10	11	9	13	5	8	12	14	4	6	2	15	7	3	1

*Three-factor interaction strings aliased with main effects are listed in Appendix A.

**Two-factor interaction strings aliased with two-factor interaction labels listed in Appendix B.

***Blank spaces represent zero percent. Spaces with zeroes in them represent some percent smaller than 1%.

modest 10% overlap may be more serious if the trend itself is very large. Rather than speculate too much on the relative magnitudes of these effects, use of the holistic design makes it possible to minimize any serious confounding trends by:

(1) Using the results of a preexperiment analysis to assign factors to the columns of the design in such a way -- to the extent possible -- that potentially large main and generalized interaction effects will be least confounded with trends.

(2) Employing statistical techniques to remove the trend components when they cannot otherwise be avoided (Simon, 1977a).

The importance of this trend-robust design is in its power and economy when used for screening purposes. The most likely sources of critical effects are kept relatively free from trend bias. It is not necessary to replicate and counterbalance the complete design to achieve this as with the conventional approach, which handles only linear trends well. Thus, a single design provides trend-robust data for a large number of factors and with a reliability that compares favorably with the more conventional few-factor-at-a-time experiments. These benefits increase as the size of the design increases (Simon, 1979).

The biggest drawback of the design is that all two-factor interactions are aliased, a maximum of eight in a single interaction string in our 2^{16-11} design (Appendix B). Other two-level Resolution IV fractional factorial designs can be found that may have less aliasing among the two-factor interactions or "aberrations patterns" (Fries and Hunter, 1978), but often show some serious trend confounding. As in the design of any experiment, the investigator must compare the advantages and disadvantages of alternative data collection plans for his particular problem at the particular stage of his investigation. The trend-robust design is intended first as an economical means of screening a very large number of factors to find the truly important ones and, as such, compares most favorably with other two-level fractional factorials [3].

If and when the empirical evidence suggests that critical interaction effects exist, the holistic approach provides an economical data collection strategy for isolating them. It also provides a means of determining whether or not the linear model of these 2^{k-p} designs adequately fits the data.

Simon (1977a, 1977b, 1981) has discussed the design, analysis, and interpretation of these trend-robust designs. Westra, Simon, Collyer, and Chambers (1981) successfully employed the holistic design to evaluate the relative importance

of 11 features of a pilot training simulator at the Naval Training Systems Center, Orlando, Florida. The basic design was used subsequently in other experiments at that facility.

SECTION III

CONCERN WITH CARRYOVER EFFECTS

E.C.Poulton at the Applied Psychology Unit, Medical Research Council, Cambridge, England, has identified a number of sources of bias in experimental data when a subject is tested across experimental conditions. Among these -- in addition to trend effects -- are range effects (1973), asymmetric transfer (1969), and others (1982) found in cognitive, perceptual-motor, vigilance, and psychophysical judgment tasks. Of particular interest here is his discussion of effects that transfer, or carryover, from one condition to another.

Unlike trend effects that tend to affect performance across conditions over the term of the experiment, carryover effects occur between adjacent conditions being studied sequentially. Their existence depends more on the nature of the task and the characteristics of the conditions being tested than on the overall amount of practice. When a subject is being tested serially across conditions, performance on one condition may affect performance on the conditions that follow. Characteristics of these specific conditions determine the nature of the carryover effect, which may be positive or negative, additive, proportional, or interactive, and may persist beyond a single trial (Finney, 1956).

Carryover effects are task specific and preexperiment analysis is needed to determine whether a particular design can adequately handle them. There are several types [4], the characteristics of which affect their degree of persistence. For example, the carryover effect may persist for only a trial or two when conditions differ in the required magnitude of a response, as would be the case when there are variations in the gear ratio of a steering wheel. Carryover may extend over a longer period when one condition involves a "natural response" (e.g., right movement of stick results in a right movement of the display indicator, which a lifetime of experience has reinforced) that is qualitatively different from the other condition, an "unnatural response" (e.g., a right movement of the stick results in a left movement of the display indicator). Principles of transfer of training frequently apply in these two types of carryover. On the other hand, there are situations in which the carryover may never dissipate once a new strategy or cognitive principle has been learned while performing on one

condition. Instead, it will persist and affect performance on all conditions for the remainder of the experiment.

Counterbalanced within-subject Latin square designs have been used by psychologists who wish to reduce the presence of individual differences in the error variance and to compensate for learning and other sequence effects. As the most primitive case, the 2 x 2 balanced Latin square has been employed, involving two groups of subjects, two conditions, and two trials (with a different condition on each trial and each group beginning with a different condition). Thus:

	Trial 1	Trial 2
Group I	A	B + a
Group II	B	A + b

The capital letters, A and B, represent direct effects while the small letters, a and b, represent the amount carried over from the previous trial and condition.

As originally conceived, a 2 x 2 Latin square is actually a two-dimensional representation of a three-dimensional space. The three independent factors are identified with the columns, rows, and diagonals. The 2 x 2 Latin square of our example is actually a fractional-factorial design, a 2^{3-2} plan with the defining contrast: [I] = ABC (see Simon, 1973 for further explanation of this symbology). The sources associated with the three degrees of freedom show the following aliasing structure: (A + BC), (B + AC), and (C + AB). Interaction ABC is lost due to the fractionation. To obtain unbiased estimates of A, B, and C, all interaction effects must be -- not merely assumed to be -- negligible.

In deciding whether it is appropriate or not to use the 2 x 2 Latin square as a within-group design as psychologists do, it helps to identify what has been confounded and to decide whether these sources are likely to exist in our experiment. If the rows (A), columns (B), and diagonals (C) are labelled Groups, Trials, and Conditions, respectively, then:

Designated Source	Actual Source of Variance
Conditions	[Conditions + (Groups x Trials)]
Groups	[Groups + (Conditions x Trials)]
Trials	[Trials + (Groups x Conditions)]

The sources within the brackets are totally aliased and cannot be isolated within the context of this design. In many

human performance experiments, these interaction terms represent the following sources of variance:

Groups x Trials:	Groups with different learning rates
Conditions x Trials:	Conditions which are learned at different rates
Groups x Conditions:	Groups with different amounts of prior experience on a condition

While experiments do differ, still it is believed that interactions of these types are found too frequently in human performance experiments to just assume them out of existence. Without the support of directly relevant empirical evidence, it would be risky to assume the additive model is the correct one. If it is not, this 2 x 2 Latin square design should not be used.

But problems with this design do not stop with these potential interactions. There are also carryover effects that can appear when a within-subject design is used. In the 2 x 2 Latin square design shown above, it can be seen that the two carryover effects, a and b, are not balanced in the design. Each does not occur with all conditions of the other dimensions; ergo, carryover effects are not orthogonal to the other sources of variance. Should there be critical carryover effects, this situation will bias the results. For screening purposes, it would be unacceptable if direct condition effects were biased.

Poulton and Freeman (1966; Poulton, 1969) noted that if amounts carried over from conditions A and B were equal -- they called this "symmetrical conditions carryover" -- then the Condition effect would not be biased. While symmetrical carryover would change the means of the two conditions, the difference between their means (i.e., the Condition effect) would be unaffected since the same amount is being added or subtracted. However, Poulton collected evidence to show that in many experiments, the size of the carryover effect differs depending on whether B follows A, or A follows B. This "asymmetrical transfer," they postulate, biases our estimate of the condition effect and seriously jeopardizes the effectiveness of within-subject designs. This, they contend, argues for the use of between-subject designs as the primary data collection method for most human performance experiments.

It helps our understanding of the problem if we identify Poulton's symmetrical and asymmetrical carryover in terms of the sources of variance in the Latin square design. Symmetrical carryover is equivalent to trial effects and asymmetrical carryover is an interactive carryover effect. In other words, when conditions change the same amount on the second trial, regardless of the conditions involved, that is a trial effect (if positive, possibly a learning effect). When, however, the

conditions show different amounts of carryover, we may have either an "additive" or an "interactive" carryover effect. In the former case, condition A -- while having a different amount of carryover than condition B -- always shows the same amount regardless of what condition follows it, while in the latter case, how much carryover A shows depends on which condition follows it. These distinctions become somewhat blurred in the oversimplified 2 x 2 Latin square, but are important to make when larger Latin squares or other counterbalanced designs are involved.

While the evidence that Poulton has accumulated shows that the 2 x 2 Latin square within-subject design as used by psychologists definitely should not be used, it does not follow that all within-subject designs should be avoided in the presence of learning and carryover.

ALTERNATIVE WITHIN-SUBJECT DESIGNS

There are different Latin squares and other balanced arrangements that can be used to offset certain carryover effects, although some are not necessarily economical nor satisfactory for our purposes (Simon, 1974). Of particular interest in this discussion, however, is the "extra-period Latin square" design (Simon, 1974, p. 43-50; 1979, p. 93-98). If an investigator wished to correct the lack of orthogonality in the 2 x 2 Latin square relevant to carryover effects, an extra-period Latin square design might be employed. This is done by giving each group a third trial, and repeating the condition in trial two, thus:

		Trials		
		1	2	3
Groups	1)	A	B + a	B + b
	2)	B	A + b	A + a

In the basic Latin square design, only one condition, B, follows condition A, and vice versa. In this design, however, conditions A and B both follow conditions A and B. Consequently, each carryover condition is paired with each direct condition an equal number of times, making these two sources of variance orthogonal. The extra-degrees of freedom obtained from the extra trials enables the additive carryover effect to be isolated. The design assumes only one-trial carryover and is not suitable if direct and carryover effects interact or, for that matter, if any of the main effects interact.

A more powerful and cleaner design when using a single subject to study the difference in performance between two conditions without concern for trend and carryover effects is

shown here. A subject would be tested on the two conditions in the sequence shown below, with the first A-condition in parentheses serving as a preconditioner to create a carryover effect for the second condition. Performance on the preconditioner would not be used in the analysis.

(A) A B B A A B B A A B B A A B B A A B B A A B B A A B B A A B B A [5]

With this plan, the estimated Condition effect, i.e., the difference between the means of conditions A and B in this sequence, will be independent of additive and interactive one-trial carryover effects as well as linear, quadratic, and cubic trend effects of any magnitude running through the data. All six sources of variance -- direct and additive and interactive carryover effects along with linear, quadratic, and cubic trends -- each with one degree of freedom, can be isolated for all practical purposes leaving a healthy error term with 25 degrees of freedom if no blocking is used. Longer and shorter designs are also available and they can be blocked if time limitations or other restrictions require it. [6]

While the above examples are interesting, they are of limited use since few real-world problems can be investigated adequately by comparing only two conditions (although it is commonly done). Even the last phase of an experimental effort, frequently the act of comparing the effectiveness of a newly chosen configuration against that of an older version, could more effectively employ a multifactor approach. There are almost always variations in the context (including the environment and operator ability) in which two real-world conditions will be operating, making the experimental question of interest: "Under what conditions is A better or worse than B?" rather than: "Is A or B better (under one experimental condition)?" The former question seeks a more generalizable answer which can be achieved only with a multifactor experiment. Later, the principles used to develop the single-factor, two-level, trend-carryover robust design shown above will be applied to the development of large multifactor holistic designs.

BETWEEN-SUBJECT DESIGN AS AN ALTERNATIVE

Concern for sequence effects in within-subject designs has led some investigators to propose that between-subject designs be used exclusively, or at least in conjunction with within-subject designs to verify their results (Poulton, 1975; 1982).

But between-subject designs have their own special problems, some of which have been noted by Poulton (1975). He writes (p. 25): "No one likes separate groups, because comparisons between conditions are confounded by differences

between the groups. To ensure a reasonable small standard error for the individual differences, you have to use large numbers of people. You are likely to need more people in each group than you would need for a complete within-subject design. But once you have accepted the need for more people, your statistical tests take care of the differences between groups. A rejection of the null hypothesis indicates that the obtained differences are not likely due to the chance allocation of people to groups."

Using many subjects, however, increases the cost of the experiment and when large, multifactor experiments are to be performed, one may run out of time, money, and available subjects. It may not be logistically possible to use a between-subject design when a great many factors are to be screened.

Furthermore, when a large number of replications (i.e., number of subjects within cells in the between-subject designs) are required to obtain a small enough standard error to overcome the effects of individual differences, reducing that problem creates another. If an investigator can afford the luxury of a large number of subjects, although in practice he may not, he will find himself in the paradoxical situation noted by Meehl (1967), that the process of replicating to increase the precision of the experimental data decreases the confidence with which one can generalize the results to the operational situation. While using many subjects per group does decrease the error variance by a factor of the square root of the quantity four over the number of replications (Simon and Westra, 1984), it also increases the chance of finding the experimental data "statistically significant" when in fact the results in practice are negligible (Bakan, 1966).

Nor can we assume that a between-subject design is devoid of sequence effects. Those created by the equipment and the environment can still be present and they can distort operator performance. If, for example, temperature shifts affect the sensitivity of some electronic sensing device that feeds information to the operator, the visual signals may vary in a systematic fashion throughout the day if proper controls are not taken, causing a comparable shift in performance among operators. Similarly, failure to allow for enough recycling time for certain devices may result in an equipment carryover effect that confounds the performance of a subject tested too soon after the previous subject has finished. Sequence effects may even be introduced by the experimenter who tests all of the subjects, e.g., as he becomes fatigued or as he gradually alters the instructions as the experiments drag on.

A more subtle weakness of the between-subject design stems from the "situation specific" nature of all human performance experiments. Results from these experiments are a function of

the characteristics present at the time data is being collected, whether studied or not, whether known or not. If the experimental subjects are not representative of the population to which the results are to be generalized, the information, e.g., which factors critically influence the performance under investigation, can be incorrect. Subject skill level is one of the more difficult experimental factors to measure accurately (Simon and Westra, 1984) and frequently little effort is made to measure it at all. The random assignment of individuals to the cells of a between-subject design does not resolve this problem and assigning all subjects at a specific skill level is difficult to do or for that matter even to ascertain that it has been done correctly. Introducing skill level as a dimension can reduce this problem but increases experimental costs considerably in large multifactor investigations. The problem is handled systematically and economically when within-subject designs are used, since a single naive subject can be tested repeatedly across all the experimental conditions, enabling the investigator to see the shift in results that takes place as ability improves with the practice.

In practice, the choice of using a within-subject or between-subjects design will probably seldom rest solely on the existence of sequence effects. Logistical considerations, such as subject availability and time and money, are also common limiting factors. Still, as Poulton has implied, the overriding principle in any experiment should be to maintain the integrity of the data. This should mean minimizing sequence effects and all other sources of irrelevant variance. It should mean including enough subjects to overcome otherwise insurmountable sampling problems. At times, however, this may best be achieved by developing new procedures that will satisfy these multiple criteria, i.e., to obtain clean, accurate, and generalizable information at minimal costs. When a large multifactor experiment is being conducted, the opportunities for meeting these criteria increase over what is possible with a few-factor experiment. A data collection plan for that purpose will be described in the next section.

SECTION IV

CARRYOVER CHARACTERISTICS OF HOLISTIC DESIGNS

To determine how resistant elements of the trend-robust holistic design are to one-trial carryover effects, correlations must be made among the main and interaction columns for direct and for additive and interactive carryover effects as well as for linear, quadratic, and cubic trends. For this analysis a 2^5 factorial design was used as the test vehicle. [7]

DIRECT AND CARRYOVER SIGN PATTERNS OF AN EXTRA PERIOD 2^k DESIGN

The 96 columns (with 32 signs to a column) required to analyze the 2^5 factorial for its trend-carryover robust characteristics are created in four sections as follows:

(1) Direct Effect Columns. In the conventional way, create the 32-row by 31-column sign matrix of estimable main and interaction direct effects for the 2^5 factorial. Columns (effects) and rows (conditions) will remain in Yates' standard order for this analysis. No Identity column is included. Plus or minus signs are used to indicate which of the two levels of each factor are found in each experimental condition (row). These arithmetic symbols, + and -, are the conventional abbreviated forms of the labels +1 and -1, respectively.

(2) Additive Carryover Columns. Create a second 32-row by 31-column sign matrix representing the additive carryover columns, one for each of the direct effects created previously. With the experimental conditions (rows) of the basic design arranged in Yates' standard order, additive carryover columns are formed by associating with each sign in the direct effects column the sign that immediately precedes it in the columnar sequence.

Since a preceding condition is needed to create a carryover effect, and since the first row of a 2^k matrix has no preceding condition, it is necessary to add an extra condition before the first row of the design, a "preconditioner." This extra condition should be the same as the one in the last row of the basic 2^5 design.

(3) Interactive Carryover Columns. Create a 32-row by 31-column sign matrix representing the interactive carryover columns. In each row, multiply the signs -- using the Rule of

Signs -- in corresponding direct and carryover columns obtained in the two previous steps.

(4) Trend Columns. Create still another 32-row by 3-column sign matrix made up of the integer Tchebycheff orthogonal polynomials (found in many statistical tables, e.g., in Beyer, 1966) representing the linear, quadratic, and cubic trends effects for $N = 2^k$ conditions of the basic design.

To illustrate the above procedures, the table below was created for just two factors, A and B, each at two levels. With two, two-level factors, a 4-row by 12-column matrix would be required, plus the "preconditioner," P, thus:

	Direct			Additive Carryover			Interactive Carryover			Trends		
	A	B	AB	A'	B'	AB'	A"	B"	AB"	L	Q	C
P)	[+	+	+]									
a)	-	-	+	+	+	+	-	-	+	-3	+1	-1
b)	+	-	-	-	-	+	-	+	-	-1	-1	+3
c)	-	+	-	+	-	-	-	-	+	+1	-1	-3
d)	+	+	+	-	+	-	-	+	-	+3	+1	+1

The extra-condition, i.e., the preconditioner, can be thought of as a warm-up trial and will not be included in the analysis. The fact that the number of conditions is not enough to provide the degrees of freedom needed to estimate all twelve effects is not relevant at this time. For the moment, we wish only to illustrate how such a table is constructed.

CORRELATIONS AMONG THE DIRECT AND CARRYOVER COLUMNS

Correlations were calculated among all 96 main and interaction columns for the 31 direct, 31 additive carryover, 31 interactive carryover, and 3 trend effects. From these correlations, the main effect columns can be selected that will create the best trend/carryover-robust design available without collecting additional data.

The actual intercolumn correlations are given in Appendix C. However, to facilitate both the discussion and interpretation of this rather large matrix, the full 93-row by 96-column intercorrelation matrix was divided into 31 x 31

submatrices and three 31 x 3 submatrices, these sections being identified by letters of the alphabet in Table 2.

TABLE 2. SECTIONING OF THE COMPLETE INTERCORRELATION MATRIX

		31 Direct	31 Additive Carryover	31 Interactive Carryover	3 Trend
R O W S	31 Direct	A	B	C	J
	31 A. Carryover	(D)	E	F	K
	31 I. Carryover	(G)	(H)	I	L

The correlations in all sections of Table 2 need not be shown since certain relationships are known on logical or other grounds. For example, correlations in Sections D, G, and H are the same as those in their counterparts B, C, and F, respectively. Only the latter will be discussed.

In Section A, i.e., the intercorrelation matrix among direct main and interaction effects of a basic 2^5 design, all correlations on the matrix diagonal are 1.ØØ, since each direct effect is correlated with itself. All off-diagonal correlations are zero, meaning there is no confounding among the direct effects, main or interaction, a result to be expected in an orthogonal factorial design. This section will not be shown.

In Section E, i.e., the intercorrelation matrix among additive carryover main and interaction effect columns, the correlations again form a unit matrix, with a diagonal of ones and all off-diagonals zero. Since the sign patterns of the additive carryover columns are the same as those of the direct columns, except that they are shifted down one row and the last row cycled to the top, this shift will not change the correlations from those found in Section A. Thus, the additive carryover effects in Section E are orthogonal to one another. This section will not be shown nor discussed.

The correlations among linear, quadratic, and cubic trend effects are not included in Table 2 since the orthogonal polynomials used to represent trend are, by definition, uncorrelated. However, how trends correlate with the direct and the additive and interactive carryover columns are shown in Table 2 and will be discussed.

CODING THE SYMBOLIC MATRICES. Omitting Sections A, D, E, G, and H for the reasons cited above leaves only Sections B, C, F, I,

J, K, and L to be considered. To facilitate the study of this rather large matrix of numbers, the correlations in the matrices were assigned to one of five categories according to size and estimated impact on the interpretation of experimental results and shown symbolically. These categories -- qualitative judgments on the part of the investigator -- were selected to represent how much a correlation within each size range is likely to distort the interpretation of experimental results prior to any effort to partial out unwanted effects. The correctness of these judgments depend, of course, not only on the size of the correlation but also on the actual and relative sizes of the effects being correlated and the practical consequences of erroneous conclusions. The distortions expected, therefore, tend to represent the worst case. It is the responsibility of the investigator to determine during the preexperiment analysis phase what conditions are likely to be expected and if they can be tolerated.

The five categories as defined below are represented by the indicated geometric symbol:

Correlation*	Proportion Overlap	Symbol	Distortion
0	.0000	□	None (N)
.0+ to .31	.00 to .09	⊡	Trivial (T)
.32 to .54	.10 to .29	⊟	Marginal (M)
.55 to .70	.30 to .49	⊠	Dangerous (D)
> .71	> .50	■	Unacceptable (U)

* Correlations with 30 degrees of freedom (N = 32) must exceed .349 to be statistically significant at the p = .05 level. In this report, correlation signs are ignored.

Symbolic representation of the intercorrelation matrices for Sections B and K, Sections C and J, and Sections I and L, are given in Tables 3-A, 3-B, and 3-C respectively. Because the intercorrelation matrix for Section F proved to be identical to that for Section C -- in size, if not in sign -- it is not printed separately. It is obtained from Table 3-B when primes are added to the direct column labels, identifying them also as additive carryover labels.

The columns in Tables 3-A, 3-B, and 3-C are identified both by their original factor labels and the new labels of a trend-robust design. The former represent the labels for all main and interaction colums of a 2^5 factorial design. The latter are the labels for the main and two-factor interaction strings of a 2^{16-11} fractional factorial design used to create the trend-robust holistic design in Table 1. Direct, additive

TABLE 3. SYMBOLIC MATRICES OF INTERCORRELATIONS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS. (2^5 factorial or equivalent fractional factorial design)

A. DIRECT vs ADDITIVE CARRYOVER (Section B = D, with K)

ADDITIVE CARRYOVER

Original Factor Labels	New Factor Labels	A'	B'	AB'	C'	AC'	BC'	ABC'	D'	AD'	BD'	ABD'	CD'	ACD'	BCD'	ABCD'	E'	AE'	BE'	ABE'	CE'	ACE'	BCE'	ABCE'	DE'	ADE'	BDE'	ABDE'	CDE'	ACDE'	BCDE'	ABCDE'
		P'	AE+'	L'	AD+'	M'	AK+'	F'	AC+'	N'	AJ+'	G'	AH+'	I'	AO+'	B'	AB+'	O'	AI+'	H'	AG+'	J'	AN+'	C'	AF+'	K'	AM+'	D'	AL+'	E'	AP+'	A'
DIRECT: A	P	U																														
B	AE+			U																												
AB	L		U																													
C	AD+				M	M	M	M																								
AC	M				M	M	M	M																								
BC	AK+				M	M	M	M																								
ABC	F				M	M	M	M																								
D	AC+								U	T	T	T	T	T	T	T																
AD	N								T	U	T	T	T	T	T	T																
BD	AJ+								T	T	T	U	T	T	T	T																
ABD	G								T	T	U	T	T	T	T	T																
CD	AH+								T	T	T	T	U	T	T	T																
ACD	I								T	T	T	T	T	U	T	T																
BCD	AO+								T	T	T	T	T	T	T	U																
ABCD	B								T	T	T	T	T	T	U	T																
E	AB+																U	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
AE	O																T	U	T	T	T	T	T	T	T	T	T	T	T	T	T	T
BE	AI+																T	T	T	U	T	T	T	T	T	T	T	T	T	T	T	T
ABE	H																T	T	U	T	T	T	T	T	T	T	T	T	T	T	T	T
CE	AG+																T	T	T	T	D	M	M	M	T	T	T	T	T	T	T	T
ACE	J																T	T	T	T	M	D	M	M	T	T	T	T	T	T	T	T
BCE	AN+																T	T	T	T	M	M	M	D	T	T	T	T	T	T	T	T
ABCE	C																T	T	T	T	M	M	D	M	T	T	T	T	T	T	T	T
DE	AF+																T	T	T	T	T	T	T	T	U	T	T	T	T	T	T	T
ADE	K																T	T	T	T	T	T	T	T	T	U	T	T	T	T	T	T
BDE	AM+																T	T	T	T	T	T	T	T	T	T	T	U	T	T	T	T
ABDE	D																T	T	T	T	T	T	T	T	T	T	U	T	T	T	T	T
CDE	AL+																T	T	T	T	T	T	T	T	T	T	T	T	D	M	M	M
ACDE	E																T	T	T	T	T	T	T	T	T	T	T	T	M	D	M	M
BCDE	AP+																T	T	T	T	T	T	T	T	T	T	T	T	M	M	M	D
ABCDE	A																T	T	T	T	T	T	T	T	T	T	T	T	M	M	D	M
TREND	L	T		T	T	T	T	T	M	T	T	T	T	T	T	T	U	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
	Q		T		T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	M	T	T	T	U	T	T	T	T	T	T	T
	C	T		T	T	T	T	T	T	T	T	T	T	T	T	T	M	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T

Notations:

- r = 0 (N)
- r = .00+ to .31 (T)
- r = .32 to .54 (M)
- r = .55 to .70 (D)
- r = >.70 (U)

TABLE 3. SYMBOLIC MATRICES OF INTERCORRELATIONS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS. (2^5 factorial or equivalent fractional factorial design) (CONTINUED)

B. INTERACTIVE CARRYOVER vs DIRECT* (Section C = G, with J)

*The correlations between interactive carryover and additive carryover effect columns (Section F = H) correspond to those shown in this table of correlations between interactive carryover and direct effect columns, respectively, except for occasional differences in signs.

DIRECT

Original Factor Labels	New Factor Labels	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD	E	AE	BE	ABE	CE	ACE	BCE	ABCE	DE	ADE	BDE	ABDE	CDE	ACDE	BCDE	ABCDE
		P	AE+	L	AD+	M	AK+	F	AC+	N	AJ+	G	AH+	I	AO+	B	AB+	O	AI+	H	AG+	J	AN+	C	AF+	K	AM+	D	AL+	E	AP+	A
INTERACTIVE CARRYOVER																																
A''	P''																															
B''	AE+''	■																														
AB''	L''	■																														
C''	AD+''	⊠	⊠	⊠																												
AC''	M''	⊠	⊠	⊠																												
BC''	AK+''	⊠	⊠	⊠																												
ABC''	F''	⊠	⊠	⊠																												
D''	AC+''	═	═	═	═	═	═	═																								
AD''	N''	═	═	═	═	═	═	═																								
BD''	AJ+''	■	•	•	•	•	•	•																								
ABD''	G''	■	•	•	•	•	•	•																								
CD''	AH+''	═	═	═	═	═	═	═																								
ACD''	I''	═	═	═	═	═	═	═																								
BCD''	AO+''	■	•	•	•	•	•	•																								
ABCD''	B''	■	•	•	•	•	•	•																								
E''	AB+''	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
AE''	O''	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
BE''	AI+''	■	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
ABE''	H''	■	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
CE''	AG+''	═	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
ACE''	J''	═	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
BCE''	AN+''	⊠	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
ABCE''	C''	⊠	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
DE''	AF+''	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
ADE''	K''	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
BDE''	AM+''	■	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
ABDE''	D''	■	•	•	•	•	•	•	•	•	•	•	•	•	•	•																
CDE''	AL+''	═	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
ACDE''	E''	═	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
BCDE''	AP+''	⊠	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
ABCDE''	A''	⊠	═	═	•	•	•	•	•	•	•	•	•	•	•	•																
TREND	L	•	•		•				═								■															
	Q			•		•	•			•	•		•					•	•		═				■							
	C	•	•		•			•	═			•		•	•		═			•		•	•			•	═		⊠			

Notations:

☐	r = 0 (N)
•	r = .00+ to .31 (T)
═	r = .32 to .54 (M)
⊠	r = .55 to .70 (D)
■	r = >.70 (U)

TABLE 3. SYMBOLIC MATRICES OF INTERCORRELATIONS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS. (2^5 factorial or equivalent fractional factorial design) (CONTINUED)

C. INTERACTIVE CARRYOVER vs INTERACTIVE CARRYOVER (Section I, with L)

		INTERACTIVE CARRYOVER																														
	Original Factor Labels	A"	B"	AB"	C"	AC"	BC"	ABC"	D"	AD"	BD"	ABD"	CD"	ACD"	BCD"	ABCD"	E"	AE"	BE"	ABE"	CE"	ACE"	BCE"	ABCE"	DE"	ADE"	BDE"	ABDE"	CDE"	ACDE"	BCDE"	ABCDE"
Original Factor Labels	New Factor Labels	P"	AE+"	L"	AD+"	M"	AK+"	F"	AC+"	N"	AJ+"	G"	AH+"	I"	AO+"	B"	AB+"	O"	AI+"	H"	AG+"	J"	AN+"	C"	AF+"	K"	AM+"	D"	AL+"	E"	AP+"	A"
A"	P"	NOT CALCULATABLE																														
B"	AE+"	NOT CALCULATABLE	U	U	D	D	D	D	M	M	U	U	M	M	U	U	T	T	U	U	M	M	D	D	T	T	U	U	M	M	D	D
AB"	L"		U	U	D	D	D	D	M	M	U	U	M	M	U	U	T	T	U	U	M	M	D	D	T	T	U	U	M	M	D	D
C"	AD+"		D	D	U	U	M	M	D	D	T	T	D	D	T	T	M	M	M	M	U	U	T	T	M	M	M	M	U	U	T	T
AC"	M"		D	D	U	U	M	M	D	D	T	T	D	D	T	T	M	M	M	M	U	U	T	T	M	M	M	M	U	U	T	T
BC"	AK+"		D	D	M	M	U	U	T	T	U	U	T	T	U	U	T	T	D	D	T	T	U	U	T	T	D	D	T	T	U	U
ABC"	F"		D	D	M	M	U	U	T	T	U	U	T	T	U	U	T	T	D	D	T	T	U	U	T	T	D	D	T	T	U	U
D"	AC+"		M	M	D	D	T	T	U	U	T	T	T	T	M	M	D	D	T	T	T	T	T	T	D	D	T	T	T	T	T	T
AD"	N"		M	M	D	D	T	T	U	U	T	T	T	T	M	M	D	D	T	T	T	T	T	T	D	D	T	T	T	T	T	T
BD"	AJ+"		U	U	T	T	U	U	T	T	U	U	M	M	M	M	T	T	U	U	T	T	D	D	T	T	U	U	T	T	D	D
ABD"	G"		U	U	T	T	U	U	T	T	U	U	M	M	M	M	T	T	U	U	T	T	D	D	T	T	U	U	T	T	D	D
CD"	AH+"		M	M	D	D	T	T	T	T	M	M	U	U	T	T	T	T	M	M	U	U	T	T	T	T	M	M	U	U	T	T
ACD"	I"		M	M	D	D	T	T	T	T	M	M	U	U	T	T	T	T	M	M	U	U	T	T	T	T	M	M	U	U	T	T
BCD"	AO+"		U	U	T	T	U	U	M	M	M	M	T	T	U	U	M	M	D	D	T	T	U	U	M	M	D	D	T	T	U	U
ABCD"	B"		U	U	T	T	U	U	M	M	M	M	T	T	U	U	M	M	D	D	T	T	U	U	M	M	D	D	T	T	U	U
E"	AB+"		T	T	M	M	T	T	D	D	T	T	T	T	M	M	U	U	T	T	T	T	M	M	T	T	T	T	M	M	T	T
AE"	O"		T	T	M	M	T	T	D	D	T	T	T	T	M	M	U	U	T	T	T	T	M	M	T	T	T	T	M	M	T	T
BE"	AI+"		U	U	M	M	D	D	T	T	U	U	M	M	D	D	T	T	U	U	T	T	M	M	T	T	U	U	T	T	U	U
ABE"	H"		U	U	M	M	D	D	T	T	U	U	M	M	D	D	T	T	U	U	T	T	M	M	T	T	U	U	T	T	U	U
CE"	AG+"		M	M	U	U	T	T	T	T	T	T	U	U	T	T	T	T	T	T	U	U	M	M	D	D	T	T	D	D	T	T
ACE"	J"		M	M	U	U	T	T	T	T	T	T	U	U	T	T	T	T	T	T	U	U	M	M	D	D	T	T	D	D	T	T
BCE"	AN+"		D	D	T	T	U	U	T	T	D	D	T	T	U	U	M	M	M	M	M	M	U	U	T	T	U	U	T	T	U	U
ABCE"	C"		D	D	T	T	U	U	T	T	D	D	T	T	U	U	M	M	M	M	M	M	U	U	T	T	U	U	T	T	U	U
DE"	AF+"		T	T	M	M	T	T	D	D	T	T	T	T	M	M	T	T	T	T	D	D	T	T	U	U	T	T	T	T	M	M
ADE"	K"		T	T	M	M	T	T	D	D	T	T	T	T	M	M	T	T	T	T	D	D	T	T	U	U	T	T	T	T	M	M
BDE"	AM+"		U	U	M	M	D	D	T	T	U	U	M	M	D	D	T	T	U	U	T	T	U	U	T	T	U	U	D	D	M	M
ABDE"	D"		U	U	M	M	D	D	T	T	U	U	M	M	D	D	T	T	U	U	T	T	U	U	T	T	U	U	D	D	M	M
CDE"	AL+"		M	M	U	U	T	T	T	T	T	T	U	U	T	T	M	M	T	T	D	D	T	T	T	T	D	D	U	U	M	M
ACDE"	E"		M	M	U	U	T	T	T	T	T	T	U	U	T	T	M	M	T	T	D	D	T	T	T	T	D	D	U	U	M	M
BCDE"	AP+"		D	D	T	T	U	U	T	T	D	D	T	T	U	U	T	T	U	U	T	T	U	U	M	M	M	M	M	M	U	U
ABCDE"	A"		D	D	T	T	U	U	T	T	D	D	T	T	U	U	T	T	U	U	T	T	U	U	M	M	M	M	M	M	U	U
TREND	L		T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
TREND	Q		T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
TREND	C		T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T

Rows: INTERACTIVE CARRYOVER

Notations:

- r = .00+ to .31 (T)
- r = .55 to .70 (D)
- r = 0 (N)
- r = .32 to .54 (M)
- r = >.70 (U)

carryover, and interactive carryover columns are distinguished by the use of primes, none for direct, one for additive carryover, and two for interactive carryover. For example, with either original or new factor labels, the letter A would represent the direct column of factor A, A' the additive carryover effect column, and A" the interactive carryover column. The same use of primes applies to interaction effect columns. When the nominal labels of an interaction column actually represents a string of interactions, the label is followed by a plus sign, e.g., AB+. The 2- and 3-factor interaction aliases for each interaction column of Table 1 are given in Appendices A and B.

Summary discussions of these tables will be given in the next section when the best columns are selected in order to create a viable trend-carryover robust (TCR) design.

SECTION V

DEVELOPING THE TREND-CARRYOVER ROBUST DESIGN

The correlation data presented in the previous section were used to develop a viable TCR design. The rational employed and the results obtained are described below. Columns were sought which would result in a reasonably orthogonal relationships among direct, additive carryover, interactive carryover, and trend effect columns. In cases where columns remained correlated to a degree, it was necessary to select the ones that would maximize the number of trivial relationships, accept as few as possible marginal relationships, and avoid dangerous and unacceptable ones. In making compromises regarding these selections, priorities were given to those favoring the orthogonality of main effects over interaction effects, direct effects over additive and interactive carryover effects, and direct with their own additive and interactive carryover effects over direct with other additive and interactive carryover effects. Tables 3-A, 3-B, and 3-C were inspected with these criteria in mind.

CORRELATIONS AMONG DIRECT COLUMNS (SECTION A) AND AMONG ADDITIVE CARRYOVER COLUMNS (SECTION E)

As indicated earlier, the intercorrelation matrix among direct columns as well as the intercorrelation matrix among additive carryover columns are both unit diagonal matrices. Each column is orthogonal to all other columns within the same matrix.

CORRELATIONS BETWEEN DIRECT AND ADDITIVE CARRYOVER COLUMNS (SECTION B)

Inspection of Table 3-A (Section B) shows that the greatest amount of nonorthogonality lies along the diagonal of the matrix. This diagonal contains the correlations between each direct column, whether main or interaction effect, and its own additive carryover counterpart. The search was made for columns that were most trend robust and had the smallest correlations between themselves and their own carryover columns.

It has been established that direct columns with an A-term in their original factor label are the most trend resistant. These columns have new factor labels A through P (see Table 1). Inspecting the diagonal values for these columns in Table 3-A,

we find that 13 out of the 31 correlations between direct and its own additive carryover column are dangerously or unacceptably high. Nine others of these diagonal pairs are correlated marginally and nine trivially, or not at all.

If we remove from consideration all direct main effect columns that show dangerously or unacceptably high correlations with their own additive carryover columns (i.e., E, I, J, K, N, O, and P), nine columns remain that tend to be relatively robust against additive carryover effects. These are direct, main effect columns with new factor labels: A, B, C, D, F, G, H, L, and M. To include their generalized interactions in the design, we must also select interaction-string columns labeled: AB+, AC+, AD+, AF+, AG+, AH+, AL+, and AM+ (see Table 1).

Further inspection of Table 3-A shows that four of these selected direct main effect columns (i.e., A, C, F, and M) are marginally correlated with their own main and certain interaction-string additive carryover columns. Direct effects A and C overlap their additive carryover counterparts only 14%; F and M overlap theirs 25%. Off-diagonal correlations are all trivial or zero except for those between M and F', M and AD+', F and AD+, and F and M', all of which are marginally correlated. Also, pairs D and AM+' and AM+ and D' are correlated at unacceptably high levels.

A more serious problem exists, however, with the direct interaction-string columns. Six out of eight of these correlate dangerously or unacceptably high with their own additive carryover interaction-string columns. What appears to be a good selection for main effects is poor insofar as their generalized interaction-strings are concerned.

Under these circumstances, if a useful TCR design is to be developed, we must be satisfied that the following statement is viable:

<u>Direct interaction-strings have no critical additive carryover effects</u>.

If this hypothesis is in fact invalid, then there would be no way to determine whether a critical interaction-string effect is a consequence of some interaction within the string or the string's additive carryover effect. In practice, however, this dilemma is probably a moot one, particularly if the design is used in a screening experiment. For one thing, critical interaction-strings in a large multifactor experiment are infrequent (though not impossible). That their carryover effects might be critical seems to be even less likely in view of their composite nature. More pertinent is the fact that in screening experiments, precise estimations of interaction effects are of less concern than the detection that they are likely to have critical effects on performance. For screening

purposes, we are interested in critical interactions only to the extent that they contain a factor that was not found to be an important main effect. This condition occurs with intrinsic interactions.

A casual check on the validity of the hypothesis may be made. One can see how the estimated effects change before and after the additive carryover effects are partialled out of the direct main and interaction effects. If the issue is still not resolved, the proper procedure would be to collect more data, a necessary step anyway to determine which interactions within a string are actually contributing to any critical string effects.

It is interesting to note that with the exception of column M, the other remaining eight columns all have an AB-term in their original factor label. Column M, new label, is column AC, original label. Furthermore, these columns (i.e., A, B, C, D, F, G, H, and L, but not M) form a complete 2 factorial.

CORRELATIONS BETWEEN DIRECT AND INTERACTIVE CARRYOVER COLUMNS (SECTION C) AND BETWEEN ADDITIVE CARRYOVER AND INTERACTIVE CARRYOVER COLUMNS (SECTION F)

The correlation matrices for Sections C and F are identical except for certain sign reversals so only one matrix is given. When developing the TCR design, magnitude rather than sign is the critical concern regarding correlations; therefore, signs are only shown in the complete correlation table in Appendix C.

Inspection of Table 3-B (representing both Sections C and F) along the diagonal reveals that between any direct column and its own interactive carryover column, the correlation is zero. This is understandable, since the cross-product between a direct column and its own interactive carryover column produces the sign pattern of their corresponding additive carryover column which has a mean of zero. Off-diagonal sign patterns reveal an interesting nonsymmetrical distribution of correlation values. Ninety-two percent of the possible 900 off-diagonal correlations are trivially related or not at all. Less than 3% are at a dangerous or unacceptable level. The remaining 5% are at marginal levels.

Off-diagonal correlations between the interactive carryover columns and the nine main effect columns under consideration, along with their eight generalized interaction-string columns, are for the most part trivial or zero. Column pairs F and F', M and M', and AD+ and AD+' are each marginally correlated (14% overlap) with two interactive carryover interaction columns, AC+" and AH+"; so are columns F', M', and AD+' of the correspondinng matrix (not shown) of Section F. Only column L of Table 3-B and column L' of the corresponding matrix of Section F show an uncomfortably large number of marginal or

dangerous correlations with nine interactive carryover main or interaction effect columns.

CORRELATIONS AMONG INTERACTIVE CARRYOVER COLUMNS (SECTION I)

A special situation exists in Table 3-C due to the fact that interactive carryover column P" (new label -- the product of columns P and P' -- contains only the minus level condition, i.e., P" = [-I], the identity column. Consequently, the "P" effect cannot be estimated. Furthermore, when an interactive carryover column containing an A-term in its original factor label is correlated with some other interactive carryover column, the correlation obtained will be the same as the corresponding column without an A-term in the label. For example, the size of the correlation between any column and ABC" will be the same as with that same column and BC".

Inspection of the selected columns -- A,B,C,D,F,G,H,L, and M, and the columns of their generalized interactions -- in Table 3-C (Section I) shows that overall, intercorrelations among the interactive carryover main effect columns are generally dangerously or unacceptably high. This is not too surprising since the two levels (plus and minus) of a great many of these interactive carryover columns are not equally represented. This less than optimum feature exists because, we must remember, we are examining the characteristics of an existing design, not one that has been created specifically to study carryover effects.

For the select group, most interactive carryover main effect columns are trivially or marginally correlated with interactive carryover interaction columns. There are exceptions. Column AM+" is correlated dangerously or unacceptably high with seven out of the nine interactive carryover main effect columns and D" is dangerously correlated with column AL+",

Too many correlations among the interactive carryover interaction columns, as a group, are dangerously or unacceptably high.

Because of the serious confounding among interactive carryover main effect columns, another limitation must be placed on the use of this TCR design, i.e.:

> Interactive carryover effects will not be estimated with this TCR design.

This does not seriously affect the use of this design as a screening tool where the primary purpose is to determine which factors are having critical effects on performance. The original reason for developing a TCR design was not to estimate carryover effects but to isolate them from direct effects as

much as possible. It does tend to accomplish this to a greater degree than more conventional designs tend to do.

CORRELATIONS OF DIRECT AND CARRYOVER COLUMNS WITH TRENDS (SECTIONS J, K, AND L).

The degree of correlation between trends and direct, additive carryover, and interactive carryover effect columns are found at the bottom of Tables 3-B, 3-A, and 3-C, respectively. Inspection reveals that while the direct main effects are at most only trivially correlated with linear, quadratic, and cubic trends, the direct interaction-strings have some very high and also marginal relationships distributed among the trends. This had already been shown in Table 1. This means that in practice one must judiciously assign the factors to the appropriate columns in a way that is likely to avoid serious confounding with trends when such effects are expected to be large. A careful preexperiment analysis will help determine if and how this can best be accomplished. Difficulties of this type can be reduced if the size of the design is increased, thereby increasing the number of available trend-robust columns. Additive carryover columns correlate with trend effects in essentially the same patterns shown by the direct columns. Interactive carryover columns are all trivially correlated with trends.

SELECTING THE FINAL TCR DESIGN

Up to this point, we have tentatively been considering the relationships among columns with new factor labels: A, B, C, D, F, G, H, L, and M to be used for direct main effects. However, in a number of sections, dangerous and unacceptable correlations occurred too frequently when columns L and M were part of the new factor label.

That fact, along with severe limitations in the total available degrees of freedom in the N = 32 design, suggests that columns L and M should not be considered main effect columns. This would leave us with a seven factor, Resolution IV design, with the experimental conditions being defined by the columns (new factor labels): A, B, C, D, F, G, and H. Each of these direct main effect columns are all ones with an AB-term in its original factor label.

The original factor label column AB itself is not included as a main effect column; it is a new factor label column L. The relevant columns for the direct interaction-string effects are: AB+, AC+, AD+, AF+, AG+, AH+, and AL+. [8]

This design is viable only if the two restrictions cited earlier are applicable, namely: (1) additive carryover interaction effects are not critical, and (2) the design is not used to estimate interactive carryover effects.

The pattern of nontrivial correlations for this seven factor (N = 32) Resolution IV, trend-carryover robust design is provided in Tables 4-A, 4-B, and 4-C. Letters M, D, and U are used to indicate the nontrivial levels -- Marginal, Dangerous, and Unacceptable, respectively. An empty space indicates that the correlation is either trivial or zero. The percent overlap for each marginal correlation follows. An investigator would use these tables to decide how best to assign experimental factors to the columns.

No table is provided for the intercorrelations among the direct columns and among the interactive carryover columns since both are diagonal matrices, i.e., with ones along the diagonal when a column is correlated with itself and zeroes off the diagonal.

SUMMARY TO TCR DESIGN CONSTRUCTION PROCEDURES

The following steps will produce a 2^{k-p} trend-carryover robust design of Resolution IV:

(1) To construct a design capable of studying k factors, each at two levels, one must use the least number of independent conditions required to satisfy this equation:

$$N = [2^x > 4(k+1)]$$

where k equals the number of factors to be studied and x equals (k-p). The reciprocal of 2^p indicates what proportion the size of this design is to the full 2^k factorial.

Example 1. To determine the size of a design needed to study seven factors, we substitute 7 for k and get 4(7+1) = 32. That makes x = 5 since 32 is equal to two to the fifth power. With k = 7, and x = 5, then 5 = (7-p) and p equals 2. Thus, this design would be of a size $1/2^2$ or one-fourth of the full 2^7 factorial. This is true, of course, since the design requires 32 conditions and a 2^7 design requires 128.

Example 2. To determine the size of a design needed to study 10 factors, we substitute 10 for k and get = 4(10+1) = 44 for which the nearest power of two equal to or greater than that number is 2^6 = 64. With k = 10 and x = 6, then p = 4. The reciprocal of 2^p, therefore, indicates that the design is a one-sixteenth fraction of the full 2^{10} factorial.

(2) Construct in Yates' standard order the sign matrix for the 2^x factorial with the value of x being determined in Step 1.

(3) Sort on A-- (original labels). That is, reorganize the columns as described earlier and illustrated in Table 1 to build a trend-robust design. At this point, the A-columns would be used for main effects and the non-A-columns for the two-factor

TABLE 4. CRITICAL RELATIONSHIPS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS IN A SEVEN FACTOR, QUARTER REPLICATE, RESOLUTION IV TREND-CARRYOVER ROBUST SCREENING DESIGN*

NOTES:

1. Direct vs Direct (Section A of Table 2) is not shown here since all direct columns are orthogonal to one another.
2. Additive Carryover vs Additive Carryover (Section E of Table 2) is not shown here since additive carryover columns are all orthogonal to one another.
3. Interaction Carryover columns in Table 4-B correlate the same (except occasionally for the sign) with both Direct and Additive Carryover columns.

NOTATIONS:

Blank space: No effect or trivial correlations (r = .00 to .31).
M: Marginal correlations (r = .32 to .54). Number with M is the percent overlap, i.e., r squared, of the two variables.
D: Dangerous correlations (r = .55 to .70).
U: Unacceptable correlations (r => .71).

ASSUMPTIONS TO BE MET:

When this design is used for screening purposes,

1. Direct interaction strings have no critical additive carryover effects.
2. Interactive carryover effects will not be estimated.

4-A	ADDITIVE CARRYOVER														TREND		
	A'	B'	C'	D'	F'	G'	H'	AB+'	AC+'	AD+'	AF+'	AG+'	AH+'	AL+'	L	Q	C
DIRECT A	M14													M14			
B																	
C			M14									M14					
D																	
F					M25					M25							
G																	
H																	
AB+								U							U		M11
AC+									U						M19		M21
AD+					M25					M25							
AF+											U					U	
AG+			M14									D				M18	
AH+													U				
AL+	M14													D			D
L								U	M19								
Q											U	M17					
C								M21									

*All labels in Tables 4-A, 4-B, and 4-C are "new factor labels".

TABLE 4. CRITICAL RELATIONSHIPS (cont'd)

4-B

		INTERACTIVE CARRYOVER														ADDITIVE CARRYOVER
		A"	B"	C"	D"	F"	G"	H"	AB+"	AC+"	AD+"	AF+"	AG+"	AH+"	AL+"	
DIRECT	A															A'
	B															B'
	C															C'
	D															D'
	F									M14				M14		F'
	G															G'
	H															H'
	AB+															AB+'
	AC+															AC+'
	AD+									M14				M14		AD+'
	AF+															AF+'
	AG+															AG+'
	AH+															AH+'
TREND	L															
	Q															
	C															

4-C

		INTERACTIVE CARRYOVER													
		A"	B"	C"	D"	F"	G"	H"	AB+"	AC+"	AD+"	AF+"	AG+"	AH+"	AL+"
INTERACTIVE CARRYOVER	A"	U	U	U	M	U	D	U				M15			M10
	B"	U	U	U	D	U	M	D	M11	M24		M11			
	C"	U	U	U	U	U	D	M	M15				M10		
	D"	M	D	U	U	D	U	U			M13			M18	D
	F"	U	U	U	D	U	U	D			M11				
	G"	D	M	D	U	U	U	U						M24	
	H"	U	D	M	U	D	U	U			M13			M18	
	AB+"		M11	M15					U	D	M				M
	AC+"		M24						D	U	D	D			
	AD+"				M13	M11		M13	M	D	U	M	U	D	U
	AF+"	M15	M11							D	M	U	D		
	AG+"			M10							U	D	U	U	D
	AH+"				M18		M24	M18			D		U	U	U
	AL+"	M10			D				M		U		D	U	U

interaction strings. As in any fractional factorial, all odd-order effects are confounded with main effects and all even-order effects are confounded with two-factor interaction strings. Any unravelling required would follow the procedures used for any screening study (Simon, 1977a).

(4) To make a design also carryover robust, sort the A-- columns into AB-- and non-AB-- groups, with the exception of column AB itself, which is to remain with the non-AB-- columns. The columns in the AB-- group can be used as main effect columns in the trend-carryover robust design.

(5) Identify the relevant two-factor interaction-string columns by finding those in which new factor label A interacts with the labels used as main effects, as well as all other columns in which the generalized interactions among main effects appear. (Column aliases will have to be worked out or derived from existing tables; see Simon, 1977a).

(6) All unused AB-- and A-- columns may be used to estimate three-factor interaction effects. All unused non-A-- columns may be used to estimate four-factor interaction effects. However, as will be discussed in the next section, we may wish to use the degrees of freedom from these unused columns to partial out trend and carryover effects, as well as to provide an estimate of internal error variance.

Just how trend and carryover robust this N = 32, Resolution IV TCR design is is summarized in Table 5. This table shows the type and frequency of nontrivial correlations among the direct columns and all other columns. The number in parenthesis associated with each cell in the table indicates the total number of correlations considered in that cell. The size of nontrivial correlations among columns are indicated by the M, D, U coding used earlier. Correlations with onesself and correlations between direct and additive carryover interactions (which our working hypothesis has excluded) are not counted in this table. Out of the total 700 correlations considered in the table, 17 (2.4%) are marginal, 3 (0.4%) are dangerous or unacceptable, and 97.2% are trivial or absent.

TABLE 5. NUMBER OF NONTRIVIAL CORRELATIONS BETWEEN DIRECT EFFECTS AND ALL SOURCES OF VARIANCE IN THIS SEVEN FACTOR TCR RESOLUTION IV DESIGN

	Direct		Additive		Interactive		Trend		
	MAIN	INT	MAIN	INT	MAIN	INT	L	Q	C
Main Direct	Ø	Ø	3-M	3-M	Ø	2-M	Ø	Ø	Ø
(N)	(42)	(49)	(49)	(49)	(49)	(49)	(21)	(21)	(21)
Interaction Direct	Ø	Ø	3-M	Ø *	Ø	2-M	1-M 1-U	1-M 1-U	2-M 1-D
(N)	(49)	(42)	(49)	(49)	(49)	(49)	(21)	(21)	(21)

* Correlations between direct and additive carryover interaction columns are not included, as per the limitation stated earlier.

Since the design is not completely orthogonal to sequence effects and is a Resolution IV design with two-factor interactions in strings, it will be necessary for the investigator to take some reasonable steps to optimize its effectiveness. First, through preexperiment analysis, he must decide whether the strengths and weaknesses of the TCR design fit the characteristics of his particular task. This frequently is not a black or white decision and involves compromise among a number of criteria. It can be done best by comparing the information, distortion, and costs associated with this design against those of alternative data collection plans. How effectively the design will be used also depends on the investigator's skill in assigning the experimental factors to the columns. Preexperiment analysis combined with the information in Table 4 and the tables in Appendix C can greatly facilitate this. It must be remembered that assigning a factor known to have a small or no carryover effect to a highly confounded column, and vice versa, tends to neutralize considerably the weaknesses from the lack of orthogonality. Finally, the design will be most effective if the investigator employs the appropriate analysis (to be discussed in the next section), properly interprets the results, and uses them to iterate the process and supplement the data, if necessary.

In an actual experiment, following the screening phase, it is likely that the above design would be amended by collecting additional data as prescribed by the holistic approach (Simon, 1977a; 1979). The purpose of adding more data might be to add new factors, shift the range of old ones, obtain more precise

information on the more important ones, isolate potentially critical effects from their aliased sources, and/or improve the fit of the response surface (if one exists).

SECTION VI

ANALYSES AND INTERPRETATIONS

Theoretically, N-1 effects can be isolated from a design composed of N independent experimental conditions. In the case of the N = 32, TCR design, 31 effects can be isolated. Because the design is in fact oversaturated with identifiable sources of variance (i.e., there are more known sources of variance than there are degrees of freedom) the experimenter must decide in which way he will cut the pie, which sources he will try to isolate. As in any oversaturated design, the remaining sources are confounded with those being isolated, introducing the specter of possible bias should they prove to have significant effects of their own. In the previous sections, a TCR design was selected that would reduce the risk of this danger. Still, the experimenter has a number of options when he analyzes the data from this design and must select the appropriate one(s) taking into consideration the information demanded by the experiment, the capability of the design to provide it, characteristics of the real world that provide _a priori_ guesses as to what might be important, and the cost of drawing wrong conclusions. If the design is primarily used as a screening design, precise estimates of the effects are less important at this stage, and ordinarily the investigator may collect additional data to resolve some ambiguities. Furthermore, the investigator is not limited to a single analysis, and in fact, if the term "research" is to be taken seriously, he will "play" with his data as a Daniel or a Tukey might do. Still, a word of caution is in order. In general, selective examination of nonorthogonal data can be dangerous. This general principle applies here and the investigator should be cautious in his interpretation.

Below, some alternative analyses are suggested for the N = 32, TCR Resolution IV design just developed. The investigator can probably think of others. The direct effect labels apply to the columns in Table 1.

ANALYSIS 1

The 31 degrees of freedom might be partitioned in the following manner:

7 MAIN EFFECTS:

Original factor labels:	All with an AB-- term in label except AB
New factor labels	A, B, C, D, F, G, H

7 TWO-FACTOR INTERACTION STRINGS:

Original factor labels	All without a B-- term in label
New factor labels	AB+, AC+, AD+, AF+, AG+, AH+, AL+

9 THREE-FACTOR INTERACTION STRINGS:

Original factor labels	Unused columns with an A-- term in label
New factor labels	E, I, J, K, L, M, N, O, P

8 FOUR-FACTOR INTERACTION STRINGS:

Original factor labels	Remaining columns, all with a B-- term in label
New factor labels	AE+, AI+, AJ+, AK+, AM+, AN+, AO+, AP+

In this partitioning, no error variance is estimated with which to do a conventional test of statistical significance. This in itself is not a serious deficiency since it is preferable to use a normal-order plot (Simon, 1977a; Daniel, 1976) to estimate the criticalness of the effects in a 2^{k-p} multifactor design of this size. Should an external estimate of error still be desired -- to use and compare with the one derived from the normal-order plot as well as to use as a test of the goodness of fit of the data -- then data might be collected at the center of the experimental space (Simon, 1977a) if the factors are quantitative and continuous and have a center point. Some investigators have been satisfied with an estimate of error variance obtained by combining the data on the four-factor interaction strings based on the assumption that it is highly unlikely that four-factor interaction effects are more than a chance occurence.

This analysis plan follows the traditional approach which fails to recognize the existence of sequence effects during the analysis of a within-subject design. It differs, however, to the extent that the TCR design is likely to be less confounded with trend and carryover effects than most available fractional factorial designs. Still, failing to remove some of the marginally confounded carryover and trend effects may create a bias and is risky, considering it is not necessary. One might wish to contrast the results from this analysis with those from the other analyses presented here insofar as the direct effects are concerned. Do the estimates of the direct effects change

markedly when sequence effects are removed? Do the changes -- if they exist -- crucially affect the interpretation of the results for screening purposes?

ANALYSIS 2

We may be content only to remove the trend effects statistically and leave whatever carryover effects that might be present confounded with the direct effects (see Table 4). We may partial trends from the experimental effects using the equations suggested by Daniel and Wilcoxon (1966) to handle linear and quadratic trends and supplemented by Simon (1977a, p. 121-128) to handle cubic trends. Since the removal of three trend effects require three additional degrees of freedom, we must sacrifice three of the 31 columns -- those which are least likely to be statistically significant -- and use them to partial out the trends. The column selected must correlate with the trend effect it represents.

Since four-factor interactions are least likely to be critical, then we might select columns D, E, and DE (original labels) to represent linear, cubic, and quadratic trends, respectively.

ANALYSIS 3

If one wished to estimate the effects of all direct (main and two-factor interaction strings), carryover (additive and interactive), and trend effects in this N = 32, TCR design, 45 degrees of freedom would be the minimum number required. Since only 31 degrees of freedom are available, some kind of compromise must be made. The goal of such a compromise would be to estimate the 31 effects most likely to be significant and ignore, i.e., allow to remain confounded, effects believed on some *a priori* basis unlikely to be critical.

This plan allows us to estimate direct effects, both main and two-factor interaction strings, and also additive carryover main effects. This might be useful where there is outside empirical evidence that interactive carryover effects are negligible.

The following (not totally independent) sources of variance would be selected to be terms in a regression analysis:

7 MAIN-DIRECT EFFECTS:
A, B, C, D, F, G, H

7 TWO-FACTOR INTERACTION DIRECT STRINGS:
AB+, AC+, AD+, AF+, AG+, AH+, AL+

7 MAIN-ADDITIVE CARRYOVER EFFECTS:
A', B', C', D', F', G', H'

9 THREE-FACTOR INTERACTION STRINGS:
E, I, J, K, L, M, N, O, P

1 FOUR-FACTOR INTERACTION STRING:
AE+

If this analysis is used, the following conditions must be valid:

1. Interactive carryover effects are negligible.

2. Interactions among additive carryover effects are negligible.

3. Direct and carryover effects do not interact critically with one another.

4. Higher-order interactions (not included in the strings listed above) are negligible.

Whatever degrees of freedom are available from noncritical three-factor interaction strings can be used to isolate trend effects or serve as an internal estimate of error variance. Some three-factor interaction strings are confounded with main effects. If from the preexperiment analysis the investigator suspects the possibility of a critical three-factor interaction, care should be taken to assign the factors so that the interaction will be located in a string that is isolated from main effects. While main effects will be robust against trend and interactive carryover, failure to isolate any component of these sequential effects that is critical will inflate the error variance and reduce the sensitivity of a significance test. Column AE+ might be selected from among the unused columns representing four-factor interaction strings to isolate the linear and cubic trends.

ANALYSIS 4

An analysis might be performed with the same breakdown as shown above, except that interactive carryover columns are used instead of the additive ones. This might be the case when there is some evidence that they are likely to have the more severe effects.

However, because of the high degree of intercorrelation among interactive carryover effects, the analysis of their individual terms should not be taken seriously. Instead, their composite variance should be evaluated for statistical significance to decide whether further isolation is needed.

ANALYSES 5A AND 5B

One may be highly selective regarding the terms (columns) that will be included in an analyses in a way that will maximize the isolation of direct effects and sacrifice the information regarding carryover effects *per se*. Information regarding carryover would be used solely to partial these effects from the direct main and interaction effects. When the TCR design is used for screening purposes, this is probably the soundest analysis of them all.

Inspection of Table 4 shows the sources of variance most likely to bias direct main and interaction effects in the N = 32, TCR design. This information is summarized here (new factor labels):

Direct Effects	Confounded Sources
A	A', AL+'
B	
C	C', AG+'
D	
F	F', AD+', AC+", AH+"
G	
H	
AB+	AB+', LT, CT
AC+	AC+', LT, CT
AD+	AD+', F', AC+", AH+"
AF+	AF+', QT
AG+	AG+', C', QT
AH+	AH+'
AL+	AL+', A', CT

The sources to serve as terms in a regression analysis are:

7 Direct main effects: A, B, C, D, F, G, H

7 Direct interaction effects: AB+, AC+, AD+, AF+, AG+, AH+, AL+

10 Additive carryover effects: A', C', F', AB+', AC+', AD+', AF+', AG+', AH+', AL+'

2 Interactive carryover effects: AC+", AH+"

3 Trend effects: LT, QT, CT

2 Residual

This plan uses 29 of the available 31 degrees of freedom for identifiable effects, leaving two for whatever purpose the experimenter deems important to isolate.

Given that plan, the investigator may approach the analysis in several ways:

A. DEVELOP A PREDICTION EQUATION. Prepare a matrix composed only of the columns for the above terms along with the performance associated with each experimental condition. Perform a multiple (possibly stepwise) regression analysis.

B. ESTIMATE DIRECT EFFECTS. Partial from each direct effect the source(s) specifically confounded with it to obtain a less confounded estimate of the particular effect (see Daniel, 1976, p. 226-232, for ways of examining partially confounded data).

The exact strategies for these analyses are not supplied since circumstances will differ and the lines to follow are best left up to the ingenuity of the investigator. There may be other more appropriate analyses for particular purposes. Those readers who have used a "cookbook" approach to experimentation in the past -- canned designs from a book and the results from a computerized analysis of variance -- may be uncomfortable with the task of having to think about the analysis and interpretation and to have to work with effects that are only quasi-orthogonal. If so, as a general model, one may find Daniel's book (1976) a breath of fresh air in this regard, as he illustrates the thought processes employed to extract useful information from slightly marred data. Where limitations on subjects and data collection time permit, a larger design might be employed to provide more trend-carryover robust columns from which to select. Here the investigator must decide whether that approach is more cost-effective than accepting some uncertainty in the first stage of the screening process and using their resources in a second stage once the location of ambiguities have been better identified.

For those who question the value of a design that requires such exercises, were they not to use a TCR design in the presence of possible sequence effects, then they must suggest an alternative plan that can examine (screen) a great many factors as economically while keeping their estimated effects robust against the possible biases. Certainly ignoring or overlooking the problem, the historical "solution" for the past 100 years, is not the answer.

SECTION VII

CONCLUSIONS

When a single subject is tested serially on all conditions of an experiment, a 2_{IV}^{k-p} holistic design can be modified in a way that will make the experimental effects robust against linear, quadratic, and cubic trends as well as additive and interactive carryover effects that may exist. Robustness relates more to the effect -- the difference between means -- than to the means themselves. This makes this design particularly useful as a screening device with which the more important factors out of a large number of potentially critical ones can be identified. Valid mean values are better obtained after the number of conditions have been markedly reduced to a relatively few, selected by the screening process. At that time, each configuration would be evaluated under operational conditions using different groups of representative subjects.

N = 64 TCR designs should be particularly attractive since they not only would allow 10 to 12 factors to be examined using a single subject with minimum concern for sequence effects, but the size is enough to provide reasonably precise and generalizable answers at marginal costs. Additional data might be required once the initial block has been collected in accordance with the principles associated with the holistic approach and screening studies (Simon, 1977a, 1977b, 1979).

The design presented in this report demonstrates that it is possible to have a practical, cost-effective TCR 2_{IV}^{k-p} holistic design. If one wished to estimate both direct and carryover effects, then totally new designs for that purpose must be developed instead of modifying existing ones as was done in this report. While such designs are likely to require that more data be collected, the additional information gained would ordinarily justify the cost.

FOOTNOTES

[1] A holistic approach to behavioral research which emphasizes the importance of accounting for as many potentially critical variables as possible, whether equipment, environment, subject, or temporal, controlled or uncontrolled, whether generated in the real world for a specific task or within the experimental setting. This approach involves a sequential strategy and bundle of techniques that enables the response surface of a large multidimensional space ot practical importance to be estimated while reducing irrelevant sources of variance, all at minimum data collection costs.

[2] When factors are aliased in 2^k or 2^{k-p} designs, the experimental conditions being compared to determine each factor's effects are identical, except for a possible reversal of signs. When it has been determined that the effect of a string composed of aliased two-factor interactions is significantly large, it is not possible to determine, without additional data collection, which interactions in the string are responsible for the observed effect. For example, the column labelled AB+ in Table 1 would actually be measuring the combined effects of a string of two-factor interactions, specifically, the combined effects of (AB + CE + DF + GH), with each term contributing an unknown amount (Appendix II). Fractional factorial designs of the same size and resolution can differ as to the number of two-factor interactions that will be confounded (Fries and Hunter, 1978). The experimenter must decide which designs best suits his needs.

[3] Some published fractional factorial designs (e.g., by the U.S. Department of Commerce, 1957) may be trend robust and have a better aberration pattern than the holistic design described here. However, some may not be trend robust and there is usually no indication either way to help the user decide. They may be used, of course, provided the experimenter first identifies what the original factor labels of the columns are (using the letters and patterns of Yates' standard order and by referencing Table 1) and does not use those too seriously confounded with trend effects.

[4] No taxonomy has ever been developed that relates tasks or task characteristics to the types and strengths of carryover and other sequence effects that are likely to exist. That would be a worthy project and one of considerable value for optimizing the selection and effective use of an experimental design.

[5] This sequence is the AB sign pattern in a conventional 2^5 factorial design arranged in Yates' standard order. Repeating the ABBA pattern for any within-subject plan of length $N = 2^k$ is the best alternative for sequencing two conditions when trends and additive and interactive carryover effects are considered potentially critical. As in the case when any experimental design is selected, the investigator must determine whether the choice is suitable for the particular task being studied. For example, the design would not be appropriate if the carryover from any early condition resulted in a permanent change throughout the remainder of the experiment since the plan is to be used only when the effects last for essentially one trial. Designs for examining more than one trial carryover effect do exist (Simon, 1974).

[6] Actually, the sources of variance are not fully isolatable. While the direct and carryover patterns for the ABBA-- pattern are orthogonal, intercorrelations between direct or carryover patterns and linear, quadratic, or cubic trends are not totally independent. There are small correlations which for all practical purposes are inconsequential and do not destroy the effectiveness of the data collection plan. Specifically, the direct pattern correlates 0.0131 with quadratic trend; this is a 0.02% overlap. The additive carryover pattern for this sequence correlates -0.1083 with linear trend and -0.1634 with cubic trend; these are 1.17% and 2.67% overlaps respectively. The interactive carryover pattern for this sequence correlates -0.0542 with linear trend and -0.0829 with cubic trend; these are 0.29% and 0.69% overlaps respectively.

[7] When the experimental conditions of a 2^k factorial design are arranged in Yates' standard order, the sign patterns for the columns of a 2^k factorial design, for example, are the same as those for any 2^k condition, two-level fractional factorial design. For example, the sign patterns of the columns of a 2^5 factorial are the same as those for a 2^{16-11}, 2^{6-1}, or 2^{13-8} fractional factorial design. The correlations among the columns are the same for all two-level factorial-like designs with 32 conditions. Rearranging the columns of one of these designs to form the trend-robust holistic design does not affect the correlation once the proper labels are identified.

[8] Note that AL+ is the only one of the interaction-string columns in which both letters in its label are not found represented among the main effect labels; L is not there. Actually, the visible letters of a string's label are only the name of the string, not necessarily the interactions of interest within the string. In the present design, adapted from that shown in Table 1, the 21 relevant two-factor interactions are aliased three to a column, including the column designated new label AL+ (see Appendix B).

NOTATIONS*

2^{k-p} design

A fractional factorial design capable of isolating the effects of K factors at two levels each using a $1/2^p$ fraction of the total design. If p equal Ø, the design is a full factorial. If a subscript, IV, is added, that indicates that the design is of Resolution IV.

A, A', A", and other letters from A through P

Used to label the columns of the experimental design. No superscript indicates a column that represents a direct effect; a single one ('), an additive carryover effect; and a double one ("), an interactive carryover effect.

A--, AB--

Represents a main or interaction term with an A or an AB in its label. For example, ACDE, AF, or ABC, ABDF, ABE, etc.

AB+, AB+', AB+", and other letters from A through P

The plus sign indicates that the designated interaction is the label of a column containing a string of interactions.

D

When referring to a correlation, indicates that its magnitude for a TCR design is "dangerously" high, i.e., from .55 and .7Ø, making the percent overlap equal to 38% to 49%. Its symbol in Table 3 is ⊠.

---

* Letters A to P are used as new factor labels for main effects in the 2^{7-2} experimental design discussed in this paper. Some of these letters are also used to represent other things. The distinction should be clear from the context.

[I]

The identity column of a 2^k factorial or fractional factorial design. All signs are plus. -[I] would be used if all signs in the column are minus.

M

When referring to a correlation, indicates that its magnitude for a TCR design is "moderately" high, i.e., from .32 to .54, making the percent overlap equal to 10% to 29%. Its symbol in Table 3 is ⊟.

T

When referring to a correlation, indicates that its magnitude for a TCR design is "trivial," i.e., from 0+ to .31, making the percent overlap equal to .00+ to .09. Its symbol in Table 3 is ⊡.

TCR design

Design that is robust against trends and carryover effects.

U

When referring to a correlation, indicates that its magnitude for TCR design is "unacceptably" high, i.e., from .71 or higher, making the percent overlap equal to 50% or higher. Its symbol in Table 3 is ■.

GLOSSARY

aliases

Term used when sources of variance in an experimental design are 100% confounded and their effects cannot be independently estimated. This aliasing occurs when, in a fractional factorial design, a contrast that estimates one effect also estimates one or more other effects.

between-subject design

Design in which a different subject or subjects are tested on each experimental condition.

carryover effect

When the characteristics of a condition affects the performance on a different condition that follows it, carryover is said to have taken place. In the literature, the terms "carryover," "crossover," and "transfer" have been used synonymously. Additive carryover occurs when the size of the carryover effect is the same for any condition whatever the condition that follows it might be. Interactive carryover occurs when the size of a carryover effect for a condition differs depending on what condition follows it.

holistic approach

A unique sequential process for ultimately mapping a large multifactor-multivariate space. The approach is composed of a philosophy, a strategy, and a bundle of techniques that require and enable an investigator to economically study under controlled conditions "all" of the relevant factors affecting performance on a specific task. Factors to be considered are from sources associated with equipment, environment, subject, or temporal conditions, as well as unwanted biases that must be reduced.

new factor labels

The labels for the experimental designs used when the sign matrix is perceived as a fractional factorial. This would be the case when it is used as a screening design.

original factor labels

The labels used when the sign matrix is perceived as a full factorial in Yates' standard order. These labels are most useful when one must mentally manipulate the design.

percent overlap

This refers to the proportion (times 100) of confounding between two columns or factors in an experimental design. Percent overlap equals the correlation squared.

Resolution IV

Term used to indicate the extent and nature of the aliasing in a fractional factorial design. The IV indicate that all main effects are independent of one another and of all two-factor interactions, and the two-factor interactions are aliased within independent two-factor interaction strings.

screening design

A fractional factorial design capable of studying the effects of a large number of factors economically under controlled conditions in order to discover which few are primarily responsible for the performance on a particular task. It is used most effectively when employed with the holistic approach.

string, interaction

A group of totally aliased interactions, the effects of which cannot be independently estimated. In this report, an interaction string is designated by a plus sign following the interaction label, e.g., AB+.

trend

An effect that progresses systematically from trial to trial through the experiment. Of interest in this report are linear (L or LT), quadratic (Q or QT), and cubic (C or CT) trends.

within-subject design

Design in which a subject is tested serially on all the conditions of an experiment.

REFERENCES

Bakan, D. The test of significance in psychological research. Psychological Bulletin, 1966, 66, 423-437.

Daniel, C. Applications of statistics to industrial experimentation. New York, NY: Wiley, 1976.

Daniel, C. and Wilcoxin, F. Factorial 2^{p-q} plans robust against linear and quadratic trends. Techometrics, 1966, 8, 258-278.

Finney, D. J. Cross-over designs in bioassay. Proceedings of the Royal Society of London, Series B, 1956, 145, 42-50.

Fries, A. and Hunter, W. G. Minimum aberration 2^{k-p} designs. Madison, WI: University of Wisconsin, Department of Statistics, Technical Report No. 551, November 1978.

Meehl, P. Theory testing in psychology and physics. Philosophy of Science, 1967, 34, 103-115.

Poulton, E. C. Bias in experimental comparisons between equipments due to the order of testing. Ergonomics, 1969, 12, 675-687.

Poulton, E. C. Unwanted range effects from using within-subject experimental designs. Psychological Bulletin, 1973, 80, 113-121.

Poulton, E. C. Range effects in experiments on people. American Journal of Psychology, 1975, 88, 3-32.

Poulton, E. C. Biases in quantitative judgments. Applied Ergonomics, 1982, 13, 31-42.

Poulton, E. C. and Freeman, P. R. Unwanted asymmetrical transfer effects with balanced experimental designs. Psychological Bulletin, 1966, 66, 496-502.

Simon, C. W. The use of central-composite designs in human factors engineering experiments. Culver City, CA: Hughes Aircraft Co., Technical Report No. AFOSR-70-6, December 1970, 52 pp. (AD 748-277)

Simon, C. W. Considerations for the proper design and interpretation of human factors engineering experiments. Culver City, CA: Hughes Aircraft Co., Technical Report No. P73-325, December 1971, 135 pp. (Microfiche)

Simon, C. W. Economical multifactor designs for human factors engineering experiments. Culver City, CA: Hughes Aircraft Co., Technical Report No. P73-326A, June 1973, 171 pp. (AD A035-108)

Simon, C. W. Methods for handling sequence effects in human factors engineering experiments. Culver City, CA: Hughes Aircraft Co., Technical Report No. P74-541A, December 1974, 197 pp. (AD A035-109)

Simon, C. W. Analysis of human factors engineering experiments: Characteristics, results, and applications. Westlake Village, CA: Canyon Research Group, Inc., Technical Report No. CWS-02-76, August 1976, 104 pp. (AD A038-184)

Simon, C. W. Design, analysis, and interpretation of screening designs for human factors engineering research. Westlake Village, CA: Canyon Research Group, Inc., Technical Report No. CWS-03-77A, September 1977a [Revised June 1978], 220 pp. (AD A056-985)

Simon, C. W. New research paradigm for applied experimental psychology: A system approach. Westlake Village, CA: Canyon Research Group, Inc., Technical Report No. CWS-04-77A, October 1977b [Revised June 1978], 123 pp. (AD A056-984)

Simon, C. W. Applications of advanced experimental methodologies to AWAVS training research. Orlando, FL: Naval Training Equipment Center, Technical Report No. NAVTRAEQUIPCEN 77-C-0065-1, January 1979, 84 pp. (AD A064-332)

Simon, C. W. Applications of advanced experimental methods to visual technology research simulator studies: Supplemental techniques. Orlando, FL: Naval Training Equipment Center, Technical Report No. NAVTRAEQUIPCEN 78-C-0060-3, January 1981, 118 pp. (AD A095-633)

Simon, C. W. and Roscoe, S. N. Application of a multifactor approach to transfer of training research. Orlando, FL: Naval Training Equipment Center, Technical Report No. NAVTRAEQUIPCEN 78-C-0060-6, July 1981, 83 pp. (AD A108-499).

Simon, C. W. and Westra, D. P. Handling bias from individual differences in between-subject holistic experimental designs. Orlando, FL: Naval Training Equipment Center, Technical Report No. NAVTRAEQUIPCEN 81-C-0105-10, 1984.

Statistical Engineering Laboratory, Fractional factorial designs for factors at two levels. Washington, DC: Government Printing Office, National Bureau of Standards Applied Mathematical Series No. 48, 1957.

Westra, D. P., Simon, C. W., Collyer, S. C., and Chambers, W.S. Simulator design features for carrier landing: I. Performance experiment. Orlando, FL: Naval Training Equipment Center Technical Report No. NAVTRAEQUIPCEN 78-C-0060-7, December 1981 (AD A122064).

APPENDIX A

MAIN AND THREE-FACTOR INTERACTION EFFECTS ALIASED IN THE 2^{16-11}_{IV} TREND RESISTANT SCREENING DESIGN IN TABLE 1.

NEW MAIN EFFECTS	ORIGINAL FACTOR LABELS	ALIASED THREE-FACTOR INTERACTION STRINGS
O	AE	ABP ACK ADJ AEH AFN AGM AIL BCN BDM BEL BFK BGJ BHI CDE GKL CFP CGI CHJ CLM DFI DGP DHK DLN EFG EIP EJK EMN FHM FJL GHN HLP IJN IKM JMP KNP
K	ADE	ABN ACO ADE AFP AGI AHJ ALM BCP BDI BEG BFO BHM BJL CDJ CEH CFN CGM CIL DFM DGN DHO DLP EFL EIN EJO EMP FGJ FHI GHP GLO HLN IJP IMO JMN NOP
N	AD	ABK ACP ADI AEG AFO AHM AJL BCO BDE BFP BGI BHJ BLM CDM CEL CFK CGJ CHI DFJ DGK DHP DLO EFH EIK EJP EMO FGM FIL GHO GLP HKL IJO IMP JKM KOP
J	ACE	ABM ACE ADO AFI AGP AHK ALN BCI BDP BEF BGO BHN BKL CDK CFM CGN CHO CLP DEH DFN DGM DIL EGL EIM EKO ENP FGK FHP FLO GHI HLM IKP INO KMN MOP
F	ABC	ABC ADL AEM AGH AIJ AKP ANO BDH BEJ BGL BIM BKO BNP CDG CEI CHL CJM CKN COP DEP DIO DJN DKM EGO EHN EKL GIP GJK GMN HIK HJP HMO ILN JLO LMP
L	AB	ABH ACG ADF AEP AIO AJN AKM BCD BEO BFG BIP BJK BMN CEN CFH CIK CJP CMO DEM DGH DIJ DKP DNO EFK EGJ EHI FIN FJO FMP GIM GKO GNP HJM HKN HOP
P	A	ABO ACN ADM AEL AFK AGJ AHI BCK BDJ BEH BFN BGM BIL CDI CEG CFO CHM CJL DEF DGO DHN DKL EIO EJN EKM FGI FHJ FLM GHK GLN HLO IJK IMN JMO KNO
E	ACDE	ABI ACJ ADK AFM AGN AHO ALP BCM BDN BFJ BGK BHP BLO CDO CFI CGP CHK CLN DFP DGI DHJ DLM FGO FHN FKL GHM GJL HIL IJM IKN IOP JKO JNP KMP MNO
I	ACD	ABE ACM ADN AFJ AGK AHP ALO BCJ BDK BFM BGN BHO BLP CDP CEF CGO CHN CKL DEG DFO DHM DJL EHL EJM EKN EOP FGP FHK FLN GHJ GLM JKP JNO KMO MNP
M	AC	ABJ ACI ADP AEF AGO AHN AKL BCE BDO BFI BGP BHK BLN CDN CFJ CGK CHP CLO DEL DFK DGJ DHI EGH EIJ EKP ENO FGN FHO FLP GIL HJL IKO INP JKN JOP
H	ABE	ABL ACD AEO AFG AIP AJK AMN BCG BDF BEP BIO BJN BKM CEK CFL CIN CJO CMP DEJ DGL DIM DKO DNP EFN EGM EIL FIK FJP FMO GIJ GKP GNO JLM KLN LOP
D	ABDE	ABG ACH AEK AFL AIN AJO AMP BCL BEN BFH BIK BJP BMO CEO CFG CIP CJK CMN EFP EGI EHJ ELM FIO FJN FKM GHL GJM GKN GOP HIM HKP HNO ILM KLO LNP
C	ABCE	ABF ADH AEJ AGL AIM AKO ANP BDL BEM BGH BIJ BKP BNO DEO DFG DIP DJK DMN EFI EGP EHK ELN FHL FJM FKN FOP GIO GJN GKM HIN HJO HMP IKL JLP LMO
A	ABCDE	BCF BDG BEI BHL BJM BKN BOP CDH CEJ CGL CIM CKO CNP DEK DFL DIN DJO DMP EFM EGN EHO ELP FGH FIJ FKP FNO GIK GJP GMO HIP HJK HMN ILO JLN KLM
B	ABCD	ACF ADG AEI AHL AJM AKN AOP CDL CEM CGH CIJ CKP CNO DEN DFH DIK DJP DMO EFJ EGK EHP ELO FGL FIM FKO FNP GIN CJO GMP HIO HJN HKM ILP JKL LMN
G	ABD	ABD ACL AEN AFH AIK AIP AMO BCH BEK BFI BIN BJO BMP CDF CEP CIO CJN CKM DEI DHL DJM DKN DOP EFO EHM EJL FIP FJK FMN HIJ HKP HNO ILM KLO LNP

APPENDIX B

TWO-FACTOR INTERACTION EFFECTS ALIASED IN THE 2^{16-11}_{IV} TREND RESISTANT SCREENING DESIGN IN TABLE 1.

	ORIGINAL FACTOR LABELS														
	E	DE	D	CE	CDE	CD	C	BE	BDE	BD	BCE	BCDE	BCD	BC	B
STRINGS OF ALIASED TWO-FACTOR INTERACTIONS (NEW LABELS)	AB	AF	AC	AG	AL	AH	AD	AI	AM	AJ	AN	AP	AO	AK	AE
	CF	BC	BF	BD	BH	BL	BG	BE	BJ	BM	BK	BO	BP	BN	BI
	DG	DL	DH	CL	CG	CD	CH	CM	CI	CE	CP	CN	CK	CO	CJ
	EI	EM	EJ	EN	DF	EO	EK	DN	DP	DO	DI	DM	DJ	DE	DK
	HL	GH	GL	FH	EP	FG	FL	FJ	EF	FI	EG	EL	EH	FP	FM
	JM	IJ	IM	IK	IO	IP	IN	GK	GO	GP	FO	FK	FN	GI	GN
	KN	KP	KO	JP	JN	JK	JO	HP	HN	HK	HM	GJ	GM	HJ	HO
	OP	NO	NP	MO	KM	MN	MP	LO	KL	LN	JL	HI	IL	LM	LP

APPENDIX C

INTERCORRELATIONS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS (2^5 factorial or 2^{16-11} fractional factorial)

A complete table of intercorrelations for a five-factor, two-levels per factor, factorial with direct, additive carryover, and interactive carryover columns would involve a 93 x 93 matrix, symmetrical about the diagonal. This matrix can be conveniently sectioned into nine 31 x 31 submatrices as shown in Table 2. However, as discussed in the body of this report, because some sections are duplicates of others and because some are unit matrices, it is only necessary to show three sections to allow a user to know the intercorrelations among all of the 93 x 93 combinations in the complete matrix. To examine the robustness of the columns against linear, quadratic, and cubic trend effects, three additional rows of correlations are added to each matrix, making it a 34 x 31 intercorrelation table. These tables -- the numerical equivalents of those in Table 3 -- are shown in this appendix.

Since the sign patterns in the columns and the relationships among the columns remain the same for equivalent-sized fractional-factorial designs, the information provided here applies to a 2^{16-11} fractional factorial design and comparable designs. The column labels are changed for that purpose and aliasing occurs. These relationships are discussed in the body of the text and by Simon (1973; 1977a; 1981).

Employing the labels and notations used in Tables 1, 3, and 4, the new factor label equivalents of the original factor labels found in this appendix are as follows:

Labels		Labels		Labels		Labels	
Orig.	New	Orig.	New	Orig.	New	Orig.	New
A	P	D	AC+	E	AB+	DE	AF+
B	AE+	AD	N	AE	O	ADE	K
AB	L	BD	AJ+	BE	AI+	BDE	AM+
C	AD+	ABD	G	ABE	H	ABDE	C
AC	M	CD	AH+	CE	AG+	CDE	AL+
BC	AK+	ACD	I	ACE	J	ACDE	E
ABC	F	BCD	AO+	BCE	AN+	BCDE	AP+
		ABCD	B	ABCE	C	ABCDE	A

The plus sign after each new label two-factor interaction indicates that it is the label of a string of aliased two-factor interactions (see Appendix B). Nothing, an apostrophe, or a quotation mark would follow each label to indicate whether the labels apply to direct, additive carryover, or interactive carryover columns, respectively.

MATRIX C-1

ADDITIVE CARRYOVER

		A'	B'	AB'	C'	AC'	BC'	ABC'	D'
DIRECT	A	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.5000	-0.5000	-0.5000	-0.5000	0.0000
	AC	0.0000	0.0000	0.0000	0.5000	-0.5000	0.5000	0.5000	0.0000
	BC	0.0000	0.0000	0.0000	0.5000	0.5000	0.5000	-0.5000	0.0000
	ABC	0.0000	0.0000	0.0000	-0.5000	-0.5000	0.5000	-0.5000	0.0000
	D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7500
	AD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	BD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	ABD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2500
	CD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	ACD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2500
	BCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2500
	ABCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	E	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	DE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ADE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TREND	LINEAR	-0.0542	0.0000	-0.1083	0.1083	-0.1083	-0.1083	-0.1083	0.3249
	QUAD	0.0000	0.0131	0.0000	0.0394	0.0131	0.0394	-0.0131	0.0919
	CUBIC	-0.0829	0.0000	-0.1634	0.1488	-0.1585	-0.1488	-0.1585	0.3097

ADDITIVE CARRYOVER

		AD'	BD'	ABD'	CD'	ACD'	BCD'	ABCD'	E'
DIRECT	A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D	-0.2500	-0.2500	-0.2500	-0.2500	-0.2500	-0.2500	-0.2500	0.0000
	AD	-0.7500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.0000
	BD	0.2500	0.2500	-0.7500	0.2500	0.2500	0.2500	0.2500	0.0000
	ABD	-0.2500	0.7500	-0.2500	-0.2500	-0.2500	-0.2500	-0.2500	0.0000
	CD	0.2500	0.2500	0.2500	0.7500	-0.2500	-0.2500	-0.2500	0.0000
	ACD	-0.2500	-0.2500	-0.2500	0.2500	-0.7500	0.2500	0.2500	0.0000
	BCD	-0.2500	-0.2500	-0.2500	0.2500	0.2500	0.2500	-0.7500	0.0000
	ABCD	0.2500	0.2500	0.2500	-0.2500	-0.2500	0.7500	-0.2500	0.0000
	E	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8750
	AE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	BE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ABE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	CE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ACE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	BCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	ABCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	DE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ADE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	BDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	ABDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	CDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	ACDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	BCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ABCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
TREND	LINEAR	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	0.7591
	QUAD	0.0394	0.0919	-0.0131	0.1970	-0.0131	-0.0131	-0.0131	0.1970
	CUBIC	-0.1488	-0.1293	-0.1488	-0.0902	-0.1293	-0.0902	-0.1683	-0.4610

ADDITIVE CARRYOVER

		AE'	BE'	ABE'	CE'	ACE'	BCE'	ABCE'	DE'
DIRECT	A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E	-0.1250	-0.1250	-0.12[illegible]0	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	AE	-0.8750	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	BE	0.1250	0.1250	-0.8750	0.1250	0.1250	0.1250	0.1250	0.1250
	ABE	-0.1250	0.8750	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	CE	0.1250	0.1250	0.1250	0.6250	-0.3750	-0.3750	-0.3750	0.1250
	ACE	-0.1250	-0.1250	-0.1250	0.3750	-0.6250	0.3750	0.3750	-0.1250
	BCE	-0.1250	-0.1250	-0.1250	0.3750	0.3750	0.3750	-0.6250	-0.1250
	ABCE	0.1250	0.1250	0.1250	-0.3750	-0.3750	0.6250	-0.3750	0.1250
	DE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.8750
	ADE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
	BDE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
	ABDE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	-0.1250
	CDE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
	ACDE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	-0.1250
	BCDE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	-0.1250
	ABCDE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
TREND	LINEAR	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083
	QUAD	0.0919	0.1970	-0.0131	0.4072	-0.0131	-0.0131	-0.0131	0.8274
	CUBIC	-0.1293	-0.0902	-0.1293	-0.0122	-0.0902	-0.0122	-0.1683	0.1439

ADDITIVE CARRYOVER

		ADE'	BDE'	ABDE'	CDE'	ACDE'	BCDE'	ABCDE'
DIRECT	A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	AE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	BE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	ABE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	CE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	ACE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	BCE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	ABCE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	DE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	ADE	-0.8750	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	BDE	0.1250	0.1250	-0.8750	0.1250	0.1250	0.1250	0.1250
	ABDE	-0.1250	0.8750	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	CDE	0.1250	0.1250	0.1250	0.6250	-0.3750	-0.3750	-0.3750
	ACDE	-0.1250	-0.1250	-0.1250	0.3750	-0.6250	0.3750	0.3750
	BCDE	-0.1250	-0.1250	-0.1250	0.3750	0.3750	0.3750	-0.6250
	ABCDE	0.1250	0.1250	0.1250	-0.3750	-0.3750	0.6250	-0.3750
TREND	LINEAR	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083
	QUAD	-0.0131	-0.0131	-0.0131	-0.0131	-0.0131	-0.0131	-0.0131
	CUBIC	-0.0122	0.1439	-0.1683	0.4561	-0.1683	-0.1683	-0.1683

MATRIX C-2* (see footnote)

		DIRECT							
		A	B	AB	C	AC	BC	ABC	D
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB*	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C*	0.5774	0.5774	-0.5774	0.0000	0.0000	0.0000	0.0000	0.0000
	AC*	-0.5774	-0.5774	0.5774	0.0000	0.0000	0.0000	0.0000	0.0000
	BC*	0.5774	-0.5774	0.5774	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC*	-0.5774	0.5774	-0.5774	0.0000	0.0000	0.0000	0.0000	0.0000
	D*	0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780	0.0000
	AD*	-0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780	0.0000
	BD*	0.7746	-0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	0.0000
	ABD*	-0.7746	0.2582	-0.2582	0.2582	-0.2582	-0.2582	0.2582	0.0000
	CD*	0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780	0.0000
	ACD*	-0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780	0.0000
	BCD*	0.7746	-0.2582	0.2582	0.2582	-0.2582	-0.2582	0.2582	0.0000
	ABCD*	-0.7746	0.2582	-0.2582	-0.2582	0.2582	0.2582	-0.2582	0.0000
	E*	0.2582	0.2582	-0.2582	0.2582	-0.2582	-0.2582	0.2582	0.2582
	AE*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	-0.2582
	BE*	0.8819	-0.1260	0.1260	-0.1260	0.1260	0.1260	-0.1260	-0.1260
	ABE*	-0.8819	0.1260	-0.1260	0.1260	-0.1260	-0.1260	0.1260	0.1260
	CE*	0.4804	0.4804	-0.4804	-0.1601	0.1601	0.1601	-0.1601	-0.1601
	ACE*	-0.4804	-0.4804	0.4804	0.1601	-0.1601	-0.1601	0.1601	0.1601
	BCE*	0.6742	-0.4045	0.4045	0.1348	-0.1348	-0.1348	0.1348	0.1348
	ABCE*	-0.6742	0.4045	-0.4045	-0.1348	0.1348	0.1348	-0.1348	-0.1348
	DE*	0.2582	0.2582	-0.2582	0.2582	-0.2582	-0.2582	0.2582	-0.2582
	ADE*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	0.2582
	BDE*	0.8819	-0.1260	0.1260	-0.1260	0.1260	0.1260	-0.1260	0.1260
	ABDE*	-0.8819	0.1260	-0.1260	0.1260	-0.1260	-0.1260	0.1260	-0.1260
	CDE*	0.4804	0.4804	-0.4804	-0.1601	0.1601	0.1601	-0.1601	0.1601
	ACDE*	-0.4804	-0.4804	0.4804	0.1601	-0.1601	-0.1601	0.1601	-0.1601
	BCDE*	0.6742	-0.4045	0.4045	0.1348	-0.1348	-0.1348	0.1348	-0.1348
	ABCDE*	-0.6742	0.4045	-0.4045	-0.1348	0.1348	0.1348	-0.1348	0.1348
TREND	LINEAR	0.0542	0.1083	0.0000	0.2166	0.0000	0.0000	0.0000	0.4332
	QUAD	0.0000	0.0000	0.0131	0.0000	0.0263	0.0525	0.0000	0.0000
	CUBIC	0.0829	0.1634	0.0000	0.3073	0.0000	0.0000	0.0098	0.4585

		DIRECT							
		AD	BD	ABD	CD	ACD	BCD	ABCD	E
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	0.0000
	AE*	0.2582	0.2582	-0.2582	0.2582	-0.2582	-0.2582	0.2582	0.0000
	BE*	0.1260	0.1260	-0.1260	0.1260	-0.1260	-0.1260	0.1260	0.0000
	ABE*	-0.1260	-0.1260	0.1260	-0.1260	0.1260	0.1260	-0.1260	0.0000
	CE*	0.1601	0.1601	-0.1601	0.1601	-0.1601	-0.1601	0.1601	0.0000
	ACE*	-0.1601	-0.1601	0.1601	-0.1601	0.1601	0.1601	-0.1601	0.0000
	BCE*	-0.1348	-0.1348	0.1348	-0.1348	0.1348	0.1348	-0.1348	0.0000
	ABCE*	0.1348	0.1348	-0.1348	0.1348	-0.1348	-0.1348	0.1348	0.0000
	DE*	0.2582	0.2582	-0.2582	0.2582	-0.2582	-0.2582	0.2582	0.0000
	ADE*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	0.0000
	BDE*	-0.1260	-0.1260	0.1260	-0.1260	0.1260	0.1260	-0.1260	0.0000
	ABDE*	0.1260	0.1260	-0.1260	0.1260	-0.1260	-0.1260	0.1260	0.0000
	CDE*	-0.1601	-0.1601	0.1601	-0.1601	0.1601	0.1601	-0.1601	0.0000
	ACDE*	0.1601	0.1601	-0.1601	0.1601	-0.1601	-0.1601	0.1601	0.0000
	BCDE*	0.1348	0.1348	-0.1348	0.1348	-0.1348	-0.1348	0.1348	0.0000
	ABCDE*	-0.1348	-0.1348	0.1348	-0.1348	0.1348	0.1348	-0.1348	0.0000
TREND	LINEAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8664
	QUAD	0.0525	0.1051	0.0000	0.2101	0.0000	0.0000	0.0000	0.0000
	CUBIC	0.0000	0.0000	0.0195	0.0000	0.0390	0.0780	0.0000	-0.3317

* The Additive vs Interactive Carryove matrix is the same as Matrix C-2 except for reversed signs in all columns with an odd number of letters in the label.

DIRECT

		AE	BE	ABE	CE	ACE	BCE	ABCE	DE
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	DE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ADE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TREND	LINEAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	QUAD	0.1051	0.2101	0.0000	0.4203	0.0000	0.0000	0.0000	0.8406
	CUBIC	0.0000	0.0000	0.0390	0.0000	0.0780	0.1561	0.0000	0.0000

DIRECT

		ADE	BDE	ABDE	CDE	ACDE	BCDE	ABCDE
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	DE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ADE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCDE*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TREND	LINEAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	QUAD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CUBIC	0.1561	0.3122	0.0000	0.6244	0.0000	0.0000	0.0000

MATRIX C-3

INTERACTIVE CARRYOVER

		A*	B*	AB*	C*	AC*	BC*	ABC*	D*
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	0.0000	1.0000	-1.0000	0.5774	-0.5774	0.5774	-0.5774	0.3780
	AB*	0.0000	-1.0000	1.0000	-0.5774	0.5774	-0.5774	0.5774	-0.3780
	C*	0.0000	0.5774	-0.5774	1.0000	-1.0000	-0.3333	0.3333	0.6547
	AC*	0.0000	-0.5774	0.5774	-1.0000	1.0000	0.3333	-0.3333	-0.6547
	BC*	0.0000	0.5774	-0.5774	-0.3333	0.3333	1.0000	-1.0000	-0.2182
	ABC*	0.0000	-0.5774	0.5774	0.3333	-0.3333	-1.0000	1.0000	0.2182
	D*	0.0000	0.3780	-0.3780	0.6547	-0.6547	-0.2182	0.2182	1.0000
	AD*	0.0000	-0.3780	0.3780	-0.6547	0.6547	0.2182	-0.2182	-1.0000
	BD*	0.0000	0.7746	-0.7746	0.1491	-0.1491	0.7454	-0.7454	-0.2928
	ABD*	0.0000	-0.7746	0.7746	-0.1491	0.1491	-0.7454	0.7454	0.2928
	CD*	0.0000	0.3780	-0.3780	0.6547	-0.6547	-0.2182	0.2182	-0.1429
	ACD*	0.0000	-0.3780	0.3780	-0.6547	0.6547	0.2182	-0.2182	0.1429
	BCD*	0.0000	0.7746	-0.7746	0.1491	-0.1491	0.7454	-0.7454	0.4880
	ABCD*	0.0000	-0.7746	0.7746	-0.1491	0.1491	-0.7454	0.7454	-0.4880
	E*	0.0000	0.2582	-0.2582	0.4472	-0.4472	-0.1491	0.1491	0.6831
	AE*	0.0000	-0.2582	0.2582	-0.4472	0.4472	0.1491	-0.1491	-0.6831
	BE*	0.0000	0.8819	-0.8819	0.3637	-0.3637	0.6547	-0.6547	0.0476
	ABE*	0.0000	-0.8819	0.8819	-0.3637	0.3637	-0.6547	0.6547	-0.0476
	CE*	0.0000	0.4804	-0.4804	0.8321	-0.8321	-0.2774	0.2774	0.3026
	ACE*	0.0000	-0.4804	0.4804	-0.8321	0.8321	0.2774	-0.2774	-0.3026
	BCE*	0.0000	0.6742	-0.6742	-0.0778	0.0778	0.8563	-0.8563	0.1529
	ABCE*	0.0000	-0.6742	0.6742	0.0778	-0.0778	-0.8563	0.8563	-0.1529
	DE*	0.0000	0.2582	-0.2582	0.4472	-0.4472	-0.1491	0.1491	0.6831
	ADE*	0.0000	-0.2582	0.2582	-0.4472	0.4472	0.1491	-0.1491	-0.6831
	BDE*	0.0000	0.8819	-0.8819	0.3637	-0.3637	0.6547	-0.6547	0.0476
	ABDE*	0.0000	-0.8819	0.8819	-0.3637	0.3637	-0.6547	0.6547	-0.0476
	CDE*	0.0000	0.4804	-0.4804	0.8321	-0.8321	-0.2774	0.2774	0.3026
	ACDE*	0.0000	-0.4804	0.4804	-0.8321	0.8321	0.2774	-0.2774	-0.3026
	BCDE*	0.0000	0.6742	-0.6742	-0.0778	0.0778	0.8563	-0.8563	0.1529
	ABCDE*	0.0000	-0.6742	0.6742	0.0778	-0.0778	-0.8563	0.8563	-0.1529
TREND	LINEAR	0.0000	0.0542	-0.0542	0.0938	-0.0938	-0.0313	0.0313	0.1433
	QUAD	0.0000	0.0000	0.0000	-0.0076	0.0076	0.0076	-0.0076	-0.0347
	CUBIC	0.0000	0.0829	-0.0829	0.1422	-0.1422	-0.0465	0.0465	0.2129

INTERACTIVE CARRYOVER

		AD*	BD*	ABD*	CD*	ACD*	BCD*	ABCD*	E*
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	-0.3780	0.7746	-0.7746	0.3780	-0.3780	0.7746	-0.7746	0.2582
	AB*	0.3780	-0.7746	0.7746	-0.3780	0.3780	-0.7746	0.7746	-0.2582
	C*	-0.6547	0.1491	-0.1491	0.6547	-0.6547	0.1491	-0.1491	0.4472
	AC*	0.6547	-0.1491	0.1491	-0.6547	0.6547	-0.1491	0.1491	-0.4472
	BC*	0.2182	0.7454	-0.7454	-0.2182	0.2182	0.7454	-0.7454	-0.1491
	ABC*	-0.2182	-0.7454	0.7454	0.2182	-0.2182	-0.7454	0.7454	0.1491
	D*	-1.0000	-0.2928	0.2928	-0.1429	0.1429	0.4880	-0.4880	0.6831
	AD*	1.0000	0.2928	-0.2928	0.1429	-0.1429	-0.4880	0.4880	-0.6831
	BD*	0.2928	1.0000	-1.0000	0.4880	-0.4880	0.4667	-0.4667	-0.2000
	ABD*	-0.2928	-1.0000	1.0000	-0.4880	0.4880	-0.4667	0.4667	0.2000
	CD*	0.1429	0.4880	-0.4880	1.0000	-1.0000	-0.2928	0.2928	-0.0976
	ACD*	-0.1429	-0.4880	0.4880	-1.0000	1.0000	0.2928	-0.2928	0.0976
	BCD*	-0.4880	0.4667	-0.4667	-0.2928	0.2928	1.0000	-1.0000	0.3333
	ABCD*	0.4880	-0.4667	0.4667	0.2928	-0.2928	-1.0000	1.0000	-0.3333
	E*	-0.6831	-0.2000	0.2000	-0.0976	0.0976	0.3333	-0.3333	1.0000
	AE*	0.6831	0.2000	-0.2000	0.0976	-0.0976	-0.3333	0.3333	-1.0000
	BE*	-0.0476	0.8783	-0.8783	0.4286	-0.4286	0.6181	-0.6181	-0.2277
	ABE*	0.0476	-0.8783	0.8783	-0.4286	0.4286	-0.6181	0.6181	0.2277
	CE*	-0.3026	0.2894	-0.2894	0.7868	-0.7868	-0.0413	0.0413	-0.1240
	ACE*	0.3026	-0.2894	0.2894	-0.7868	0.7868	0.0413	-0.0413	0.1240
	BCE*	-0.1529	0.5919	-0.5919	-0.2548	0.2548	0.8704	-0.8704	0.3830
	ABCE*	0.1529	-0.5919	0.5919	0.2548	-0.2548	-0.8704	0.8704	-0.3830
	DE*	-0.6831	-0.2000	0.2000	-0.0976	0.0976	0.3333	-0.3333	-0.0667
	ADE*	0.6831	0.2000	-0.2000	0.0976	-0.0976	-0.3333	0.3333	0.0667
	BDE*	-0.0476	0.8783	-0.8783	0.4286	-0.4286	0.6181	-0.6181	0.2928
	ABDE*	0.0476	-0.8783	0.8783	-0.4286	0.4286	-0.6181	0.6181	-0.2928
	CDE*	-0.3026	0.2894	-0.2894	0.7868	-0.7868	-0.0413	0.0413	0.5375
	ACDE*	0.3026	-0.2894	0.2894	-0.7868	0.7868	0.0413	-0.0413	-0.5375
	BCDE*	-0.1529	0.5919	-0.5919	-0.2548	0.2548	0.8704	-0.8704	-0.1741
	ABCDE*	0.1529	-0.5919	0.5919	0.2548	-0.2548	-0.8704	0.8704	0.1741
TREND	LINEAR	-0.1433	-0.0419	0.0419	-0.0205	0.0205	0.0699	-0.0699	0.2097
	QUAD	0.0347	0.0237	-0.0237	0.0248	-0.0248	-0.0170	0.0170	-0.1187
	CUBIC	-0.2129	-0.0598	0.0598	-0.0267	0.0267	0.1039	-0.1039	0.2991

INTERACTIVE CARRYOVER

		AE*	BE*	ABE*	CE*	ACE*	BCE*	ABCE*	DE*
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	-0.2582	0.8819	-0.8819	0.4804	-0.4804	0.6742	-0.6742	0.2582
	AB*	0.2582	-0.8819	0.8819	-0.4804	0.4804	-0.6742	0.6742	-0.2582
	C*	-0.4472	0.3637	-0.3637	0.8321	-0.8321	-0.0778	0.0778	0.4472
	AC*	0.4472	-0.3637	0.3637	-0.8321	0.8321	0.0778	-0.0778	-0.4472
	BC*	0.1491	0.6547	-0.6547	-0.2774	0.2774	0.8563	-0.8563	-0.1491
	ABC*	-0.1491	-0.6547	0.6547	0.2774	-0.2774	-0.8563	0.8563	0.1491
	D*	-0.6831	0.0476	-0.0476	0.3026	-0.3026	0.1529	-0.1529	0.6831
	AD*	0.6831	-0.0476	0.0476	-0.3026	0.3026	-0.1529	0.1529	-0.6831
	BD*	0.2000	0.8783	-0.8783	0.2894	-0.2894	0.5919	-0.5919	-0.2000
	ABD*	-0.2000	-0.8783	0.8783	-0.2894	0.2894	-0.5919	0.5919	0.2000
	CD*	0.0976	0.4286	-0.4286	0.7868	-0.7868	-0.2548	0.2548	-0.0976
	ACD*	-0.0976	-0.4286	0.4286	-0.7868	0.7868	0.2548	-0.2548	0.0976
	BCD*	-0.3333	0.6181	-0.6181	-0.0413	0.0413	0.8704	-0.8704	0.3333
	ABCD*	0.3333	-0.6181	0.6181	0.0413	-0.0413	-0.8704	0.8704	-0.3333
	E*	-1.0000	-0.2277	0.2277	-0.1240	0.1240	0.3830	-0.3830	-0.0667
	AE*	1.0000	0.2277	-0.2277	0.1240	-0.1240	-0.3830	0.3830	0.0667
	BE*	0.2277	1.0000	-1.0000	0.5447	-0.5447	0.4927	-0.4927	0.2928
	ABE*	-0.2277	-1.0000	1.0000	-0.5447	0.5447	-0.4927	0.4927	-0.2928
	CE*	0.1240	0.5447	-0.5447	1.0000	-1.0000	-0.3239	0.3239	0.5375
	ACE*	-0.1240	-0.5447	0.5447	-1.0000	1.0000	0.3239	-0.3239	-0.5375
	BCE*	-0.3830	0.4927	-0.4927	-0.3239	0.3239	1.0000	-1.0000	-0.1741
	ABCE*	0.3830	-0.4927	0.4927	0.3239	-0.3239	-1.0000	1.0000	0.1741
	DE*	0.0667	0.2928	-0.2928	0.5375	-0.5375	-0.1741	0.1741	1.0000
	ADE*	-0.0667	-0.2928	0.2928	-0.5375	0.5375	0.1741	-0.1741	-1.0000
	BDE*	-0.2928	0.7460	-0.7460	0.2219	-0.2219	0.7645	-0.7645	-0.2277
	ABDE*	0.2928	-0.7460	0.7460	-0.2219	0.2219	-0.7645	0.7645	0.2277
	CDE*	-0.5375	0.2219	-0.2219	0.5897	-0.5897	0.0216	-0.0216	-0.1240
	ACDE*	0.5375	-0.2219	0.2219	-0.5897	0.5897	-0.0216	0.0216	0.1240
	BCDE*	0.1741	0.7645	-0.7645	0.0216	-0.0216	0.7091	-0.7091	0.3830
	ABCDE*	-0.1741	-0.7645	0.7645	-0.0216	0.0216	-0.7091	0.7091	-0.3830
TREND	LINEAR	-0.2097	-0.0478	0.0478	-0.0260	0.0260	0.0803	-0.0803	-0.0140
	QUAD	0.1187	0.0579	-0.0579	0.0652	-0.0652	-0.0549	0.0549	0.0712
	CUBIC	-0.2991	-0.0624	0.0624	-0.0277	0.0277	0.1128	-0.1128	-0.0082

INTERACTIVE CARRYOVER

		ADE*	BDE*	ABDE*	CDE*	ACDE*	BCDE*	ABCDE*
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	-0.2582	0.8819	-0.8819	0.4804	-0.4804	0.6742	-0.6742
	AB*	0.2582	-0.8819	0.8819	-0.4804	0.4804	-0.6742	0.6742
	C*	-0.4472	0.3637	-0.3637	0.8321	-0.8321	-0.0778	0.0778
	AC*	0.4472	-0.3637	0.3637	-0.8321	0.8321	0.0778	-0.0778
	BC*	0.1491	0.6547	-0.6547	-0.2774	0.2774	0.8563	-0.8563
	ABC*	-0.1491	-0.6547	0.6547	0.2774	-0.2774	-0.8563	0.8563
	D*	-0.6831	0.0476	-0.0476	0.3026	-0.3026	0.1529	-0.1529
	AD*	0.6831	-0.0476	0.0476	-0.3026	0.3026	-0.1529	0.1529
	BD*	0.2000	0.8783	-0.8783	0.2894	-0.2894	0.5919	-0.5919
	ABD*	-0.2000	-0.8783	0.8783	-0.2894	0.2894	-0.5919	0.5919
	CD*	0.0976	0.4286	-0.4286	0.7868	-0.7868	-0.2548	0.2548
	ACD*	-0.0976	-0.4286	0.4286	-0.7868	0.7868	0.2548	-0.2548
	BCD*	-0.3333	0.6181	-0.6181	-0.0413	0.0413	0.8704	-0.8704
	ABCD*	0.3333	-0.6181	0.6181	0.0413	-0.0413	-0.8704	0.8704
	E*	0.0667	0.2928	-0.2928	0.5375	-0.5375	-0.1741	0.1741
	AE*	-0.0667	-0.2928	0.2928	-0.5375	0.5375	0.1741	-0.1741
	BE*	-0.2928	0.7460	-0.7460	0.2219	-0.2219	0.7645	-0.7645
	ABE*	0.2928	-0.7460	0.7460	-0.2219	0.2219	-0.7645	0.7645
	CE*	-0.5375	0.2219	-0.2219	0.5897	-0.5897	0.0216	-0.0216
	ACE*	0.5375	-0.2219	0.2219	-0.5897	0.5897	-0.0216	0.0216
	BCE*	0.1741	0.7645	-0.7645	0.0216	-0.0216	0.7091	-0.7091
	ABCE*	-0.1741	-0.7645	0.7645	-0.0216	0.0216	-0.7091	0.7091
	DE*	-1.0000	-0.2277	0.2277	-0.1240	0.1240	0.3830	-0.3830
	ADE*	1.0000	0.2277	-0.2277	0.1240	-0.1240	-0.3830	0.3830
	BDE*	0.2277	1.0000	-1.0000	0.5447	-0.5447	0.4927	-0.4927
	ABDE*	-0.2277	-1.0000	1.0000	-0.5447	0.5447	-0.4927	0.4927
	CDE*	0.1240	0.5447	-0.5447	1.0000	-1.0000	-0.3239	0.3239
	ACDE*	-0.1240	-0.5447	0.5447	-1.0000	1.0000	0.3239	-0.3239
	BCDE*	-0.3830	0.4927	-0.4927	-0.3239	0.3239	1.0000	-1.0000
	ABCDE*	0.3830	-0.4927	0.4927	0.3239	-0.3239	-1.0000	1.0000
TREND	LINEAR	0.0140	0.0614	-0.0614	0.1127	-0.1127	-0.0365	0.0365
	QUAD	-0.0712	-0.0347	0.0347	-0.0526	0.0526	0.0443	-0.0443
	CUBIC	0.0082	0.0876	-0.0876	0.1629	-0.1629	-0.0477	0.0477

END

DTIC

5-86